Current Sheet Permeability in Electromagnetic Pulsed Plasma Thrusters*

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A case is made for the existence of permeability of current sheets in electromagnetic pulsed plasma thrusters. Permeability refers to the leakage of plasma or neutral particles behind the sheet and could cause a significant effect on the performance of the thruster. Analytical calculations are presented which demonstrate the mismatch between experimental velocity profiles and theoretical snowplow performances. Electron density measurements show a significant plasma wake following the sheet. Arguments are also presented based on these data, that in some cases the accelerating plasma sheet entrains less than one third of the propellant. An index of permeability is defined as a ratio of sheet-trailing mass to encountered propellant mass. It may be quantified either by directly measuring the trailing mass, or by measuring the mass entrained by the sheet. The diagnostics needed to measure this index are discussed.

I. INTRODUCTION

The study presented in this paper is concerned with the issue of current sheet permeability in electromagnetic pulsed plasma thrusters (PPTs). Permeability refers to the leakage or penetration of plasma or neutrals under or through the current sheet. An electromagnetic PPT is one in which the electromagnetic acceleration dominates over the electrothermal effects, as in the case of most gas-fed PPTs and some ablative PPTs. In such electromagnetic PPTs, the snowplow model of a current sheet has provided much insight as an ideal case [1]. In this model all the neutrals in front of the sheet are ionized and entrained by it. Experiments show that this ideal situation is often not the case.

Permeability could lead to inefficiencies in acceleration. Thrust and propellant utilization efficiency will decrease if a significant percentage of the propellant gas is not accelerated to current sheet speeds.

This problem has been identified by researchers in the past. Burkhardt and Lovberg [2] used circumstantial evidence to argue the existence of current sheet permeability. They noted an increase in speed of the sheet and a decrease in canting angle. Both effects could be explained by current sheet permeability to gas ahead of it. Later, Lovberg [3] also noticed plasma lagging along the cathode after the current sheet had passed.

York and Jahn [4] found the same plasma wake in their studies of pinch devices. They used pressure measurements to identify the "slow-moving" wake, but they did not characterize the changes in the wake with varying parameters.

More recently Markusic [5], while studying current sheet canting on the same device employed in our work, found evidence of a plasma wake as well through photographic and interferometric measurements. Markusic's work suggests that current sheet canting may cause plasma to be swept under the sheet at the cathode and be left behind.

In addition to this wake of mass that is believed to be swept *under* the sheet at the cathode, ions and neutrals may also be permeating *through* the sheet. At this time, we will not make a distinction between these leakages except to say that leakage through the sheet may occur due to non-uniformities or instabilities in the sheet while the leakage under the sheet could occur with a uniform, but canted, sheet.

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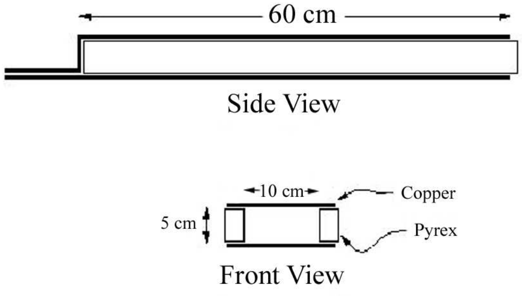

FIG. 1: Schematic of the test accelerator.

In section III an analysis is carried out that compares the theoretical snowplow velocity development to that obtained by previous experiments. An experimental exploration of the plasma density in the current sheet is described in section IV. In section V an index of permeability is defined and discussed, from which the plasma parameters required to define dependencies of the index become clear. The necessary plasma measurements are defined in section VI to guide future investigation of this problem.

II. DESCRIPTION OF THE APPARATUS

The device used in this study is the same as that used by Markusic in his current sheet canting studies. It is described in detail elsewhere [5], but will be briefly described here as well. The accelerator was not meant to be an operating thruster, but was constructed with ease of diagnosis in mind. It is a parallel plate accelerator with glass sidewalls, providing direct optical access to the discharge. The anode and cathode are made of copper and the area of the acceleration region measures 60 cm long and 10 cm wide, with a gap of 5 cm between the plates. A uniform gas fill inside the accelerator and tank was used for all experiments to avoid the complicating effects of propellant injection non-uniformities.

This accelerator was placed in a plexi-glass vacuum tank facility for testing. This facility, described in detail by Jahn and Clark [6], is 0.9 m in diameter and 1.8 m long.

The current sheet is formed in this device through a pulse forming network consisting of ten stages of 10 μF capacitors and 100 nH inductors. The device is triggered by an ignitron. A typical current pulse resulting from a 9 kV applied voltage is a fairly flat current of about 60 kA lasting approximately 25 μs, with a rise time on the order of 2 μs.

III. COMPARISON BETWEEN IDEAL SNOWPLOW AND EXPERIMENT

Previous work [5] shows that the current sheet in a pulsed plasma thruster (in particular, the same test accelerator used in this study) behaves in a highly non-ideal manner. The current sheet is seen to cant to a high angle and plasma was seen to leak behind the sheet on the cathode. Clearly the reality of operation of these devices is far from the ideal snowplow model. It is insightful, however, to make a first order comparison between the ideality and the reality.

The snowplow model of gas-fed pulsed plasma thruster operation is as follows. The equation of motion of the current sheet can be written [7]:

$$\frac{d(mv)}{dt} = m\frac{d^2x}{dt^2} + \frac{dm}{dt}\frac{dx}{dt} = \frac{1}{2}L'J^2 \tag{1}$$

where $m(t)$ is the mass in the sheet, $v(t)$ is the sheet velocity, $x(t)$ is the position of the sheet along the rails, L' is the inductance per unit length of the accelerator, and J is the current. The term on the right hand side of the equation is the familiar Lorentz force. The inductance per unit length of a parallel plate accelerator is given by $L' = 0.306\mu_0$ for our geometry [8]. For the present situation, the mass is initially evenly distributed in the accelerator, and in the ideal case all of the mass is picked up by the sheet, thus $m(t) = m_0 + hd\rho x$. Here h and d are the height and width of the channel, respectively, and ρ is the density of the propellant. The quantity m_0 is the initial mass of the sheet which, while difficult to estimate, has little impact on our argument as will be shown shortly. Another approximation that will be made is that the current, J is constant for all time, which is valid for the operating conditions of our experiment [5].

The experimental parameters are given in table I. The value for m_0 is approximately taken from an the initial mass in the first 0.5 cm of the accelerator.

Solving for $x(t)$ and $v(t)$ with $x(0) = 0$ and $v(0) = 0$, we find:

$$x = \frac{1}{hd\rho}[-m_0 + \sqrt{m_0^2 + \frac{1}{2}hd\rho L'J^2t^2}] \tag{2}$$

$$v = \frac{\frac{1}{2}L'J^2 \times t}{\sqrt{(m_0^2 + \frac{1}{2}hd\rho L'J^2t^2)}} \tag{3}$$

We can immediately see that as time increases under the constant current case, the initial mass only

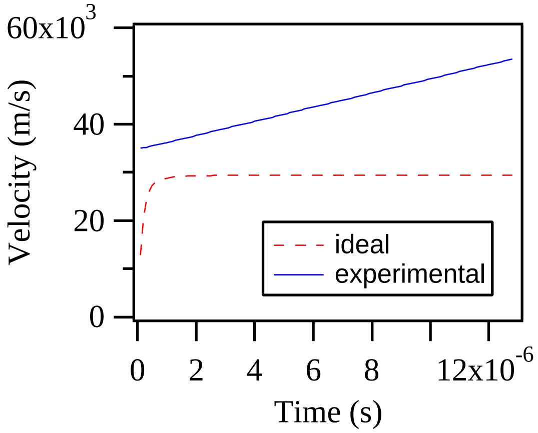

FIG. 2: Comparison between the theoretical snowplow velocity profile derived from equation 3 to the experimentally measured profile from Markusic [5] and expressed in equation 5.

$hd(m^2)$	$\rho(kg/m^3)$	$m_0(kg)$	$L'(H/m)$	$J(A)$
5.16×10^{-3}	1.6×10^{-4}	5×10^{-9}	3.845×10^{-7}	6.1×10^{4}

TABLE I: Experimental parameters for Argon, 75 mTorr discharge.

has an effect on the initial transient of the velocity and the velocity asymptotes to a constant value, given by:

$$v=\sqrt{\frac{\frac{1}{2}L'J^2}{hd\rho}} \tag{4}$$

Experimental findings, however, fail to follow this general trend. Instead, it has been observed by Markusic [5] that the velocity increases at a fairly steady rate as the current sheet travels down the rails. Specifically, for a case of Argon propellant at a uniform fill pressure of 75 mTorr and under an approximately constant current, the current sheet is found, within the error bars, to follow the trajectory:

$$v=cx+b \tag{5}$$

where $c=5\times10^4 s^{-1}$ and $b=3.5\times10^4\frac{m}{s}$.

It is useful to compare the experimental velocity profile with the ideal snowplow velocity profile. This is done in figure 2.

It is also insightful to consider the mass profile that the experimental sheet would sweep if it was indeed a perfect current sheet. In other words, since it is clear that the sheet is not sweeping all of the gas as a snowplow, let us consider what fraction of the gas it *is* sweeping (if that fraction of gas was swept perfectly). The effective mass of propellant entrained by the sheet can be estimated by using the experimental velocity history (eqn. 5) in the snowplow equation (1), and solving for $m(t)$:

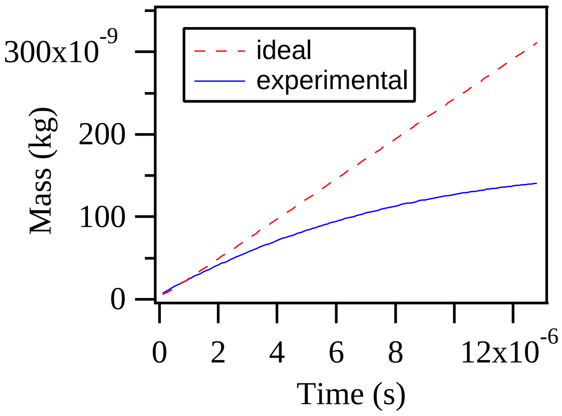

FIG. 3: Comparison of the theoretical snowplow mass accumulation (equation 7) to the experimentally implied mass accumulation (equation 6) corresponding to the experimentally measured velocity profile from equation 5.

$$m(t)=(\frac{\frac{1}{2}L'J^2}{b}t+m_0)e^{-ct} \tag{6}$$

By comparison, the ideal snowplow current sheet $m(t)$ is:

$$m(t)=\sqrt{(m_0^2+\frac{1}{2}hd\rho L'J^2t^2)} \tag{7}$$

Figure 3 shows these two profiles. It is easy to see that the effective mass entrained by the current sheet as inferred from the experiment is increasingly less than that of the ideal sheet. We also see that the velocity of the experimental current sheet is greater than expected. These effects can be explained if the current sheet is permeable.

IV. ELECTRON NUMBER DENSITY MEASUREMENTS

Electron number density measurements in the sheet as well as trailing the sheet were conducted in an effort to quantify the current sheet permeability problem. Photographic and previous interferometric evidence suggests that plasma may be forced

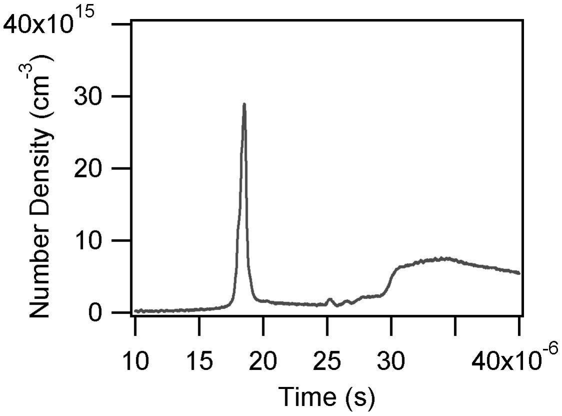

FIG. 4: Electron number density vs. time at 38 cm from the breech, halfway between anode and cathode. The discharge conditions are: Argon, 150 mTorr, 9kV.

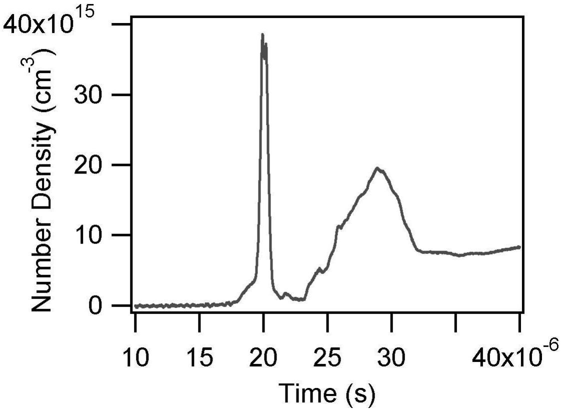

FIG. 5: Electron number density vs. time at the same point and conditions as figure 4, but here the pressure is 250 mTorr

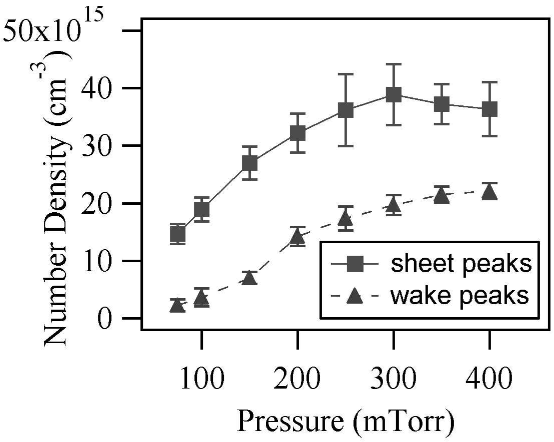

FIG. 6: Comparison of the peak electron number densities in the current sheet and in the plasma wake over a range of Argon pressures. The discharge was operated at 9kV.

under the canted sheet at the cathode, forming a long trail mostly in the vicinity of the cathode [9]. This trailing plasma has yet to be characterized systematically.

Measurements were made with a quadrature heterodyne interferometer identical to the one employed by Markusic [9]. Measurements were taken at a point equidistant from the anode and cathode, and at 38 cm from the breech of the accelerator. Two typical measurements at two pressures (150 mTorr and 250 mTorr) are shown in figures 4 and 5.

The current sheet is seen to pass the point of measurement quickly, on the order of 3 μs, implying it remains relatively tightly compressed spatially. After the current sheet passes, however, the electron number density rises again slowly, implying a wake of plasma has been left behind. The speed of this wake is not known, thus from these data alone we cannot determine its physical dimension. Photographic evidence, however, shows the wake to be significantly broader than the sheet [9].

To quantify the significance of the plasma wake, we compared the peaks in the electron number density of the sheet and the wake for various discharge conditions. Argon discharges were employed, at eight pressures, from 75 to 400 mTorr, and at two different voltage (current) levels. These data are presented in figures 6 and 7. Each point is the result of ten trials at identical conditions, and the error bars resulting from the standard deviations of the trials are shown.

This study, though limited in its scope, shows many interesting conclusions. First, it is apparent that at all pressures and both voltages tested, the plasma wake is of significant density compared to the current sheet. The peak electron number densities are comparable, especially at higher pressures. In fact, considering that the spatial extent of the wake may be larger than the sheet, the former may contain more particles than the later.

Secondly, as the pressure is increased, the sheet appears to be able to entrain a lower fraction of the particles. That is, fewer neutral particles in front of the sheet are turned into ionized particles within the sheet. This is not unexpected. As the neutral density in front of the sheet increases it becomes more difficult to ionize a large percentage of the propellant. At the same time, however, the wake continues to pick up a significant amount of charged particles. No conclusions can be made about the possible leakage of neutral particles, however it can be imagined

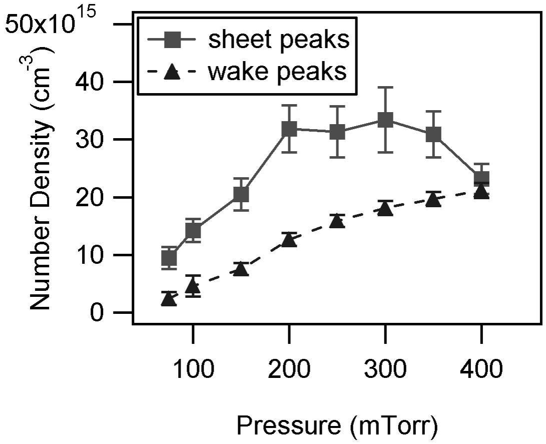

FIG. 7: Comparison of the peak electron number densities in the current sheet and in the plasma wake over a range of Argon pressures. The discharge was operated at 8kV.

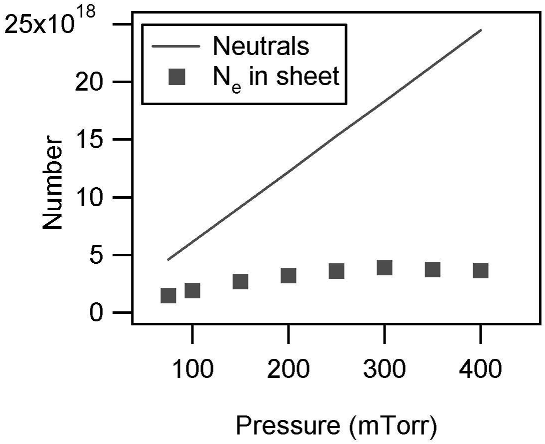

FIG. 8: Comparison of the number of neutrals encountered by the current sheet up to the measurement point, to an approximate count of the number of electrons in the sheet at that point.

that this is a problem worthy of study as well. Also, it is not clear whether the sheet attains a certain level of plasma density quickly and then maintains it by letting neutrals and plasma permeate the sheet, or continues to build mass as it propagates. Figure 3, however, implies that the sheet builds mass more slowly than expected and the accelerated mass reaches an asymptote.

Another conclusion can be drawn from the measurements of electron number density in the current sheet. If we take as an example data shown in figure 4 and assume a physical current sheet width of approximately 2 cm, which is justified by other observations [5], we can estimate the total number of electrons in the sheet. If we assume singly ionized ions, and that the peak level measured in figure is constant in the 2 cm × 5 cm × 10 cm sheet volume, we will have an overestimate. For the example above, this turns out to be 2.7×10^{18} ions in the sheet. If we compare this to the amount of neutrals that the sheet should have encountered by that point in its life, we will have a rough estimate of the percentage of neutrals that are ionized in (and remain in) the sheet. This number turns out to be 9.2×10^{18} neutrals, giving about 30% ionization. Figure 8 shows this calculation carried out for the entire pressure range of the above interferometry data (no error bars are shown due to the approximate nature of this calculation).

We note that according to this rough estimation, the percentage of neutrals that is converted to ions in the sheet is low, and decreases with increasing pressure. This reiterates our previous comments about the decreasing ability of the sheet to entrain mass at higher pressures. This experimental evidence along with the analytical discrepancies noted above leads us to the conclusion that significant permeability exists in electromagnetic pulsed plasma thruster current sheets. The problem, however, remains to be quantified and probed in a systematic manner.

V. AN INDEX OF PERMEABILITY

In his analysis of efficiency of gas-fed pulsed plasma thrusters, Ziemer [10] identifies a current sheet sweeping efficiency as the mass that is accelerated divided by the available mass. This is useful from a propulsive standpoint, as it describes the fraction of the propellant which contributes to thrust.

This index is slightly different from what we are concerned with, however. Mass that is trailing the current sheet is said to have permeated the sheet, but it is still possible for that mass to travel with significant momentum. An open research question that we intend to study in the future is how quickly the trailing plasma and trailing neutrals are moving. We also wish to have an understanding of what fraction of mass is contained in the sheet and what fraction is left behind. While a rough measure of this can be taken from data such as in figures 4 and 5, these measurements are taken at single physical points. We wish to have an understanding of the permeability of the current sheet as a function of time while varying parameters such as atomic mass of the propellant, current, and pressure.

Thus, as an index of permeability it is useful to consider the mass that is left behind the current sheet compared to the mass that the sheet has encountered to that point. We will call this trailing mass m_{trail}. The mass of the neutrals that was in the volume cleared by the sheet will be labeled m_{avail} for available mass. Thus, the index of permeability is quite simply:

$$\phi(t) = \frac{m_{\text{trail}}(t)}{m_{\text{avail}}(t)} \tag{8}$$

Note the time dependence of each term. The mass left behind increases, and the mass of neutrals that is encountered by the sheet increases as well. Also note that $\phi = 0$ indicates a non-permeable sheet, and $\phi = 1$ indicates a completely permeable sheet.

The denominator of this index is straightforward; the mass initially in the volume behind the sheet is just the ambient fill density times the volume.[11] Also if the sheet is canted with no discontinuity of angle, the volume can be expressed in terms of the velocity of the middle of the sheet (halfway between anode and cathode). The denominator term becomes:

$$m_{\text{avail}}(t) = hd\rho \int_0^t v(t)dt \tag{9}$$

If the velocity profile is not known, this term can still be calculated at a given time with knowledge of the sheet's position at that time.

The numerator of the expression for index of permeability is more difficult. The mass that is left behind the sheet may be made up of both ions and neutrals. Measuring ion density through interferometry is possible with current diagnostics, as we have shown above. Currently we do not have the capability to measure neutral densities, although such a measurement is a future goal of this study. Through either fast dynamic pressure measurements, or two-color interferometry we hope to infer the neutral density behind the sheet. Direct measurements of neutral and ion densities behind the sheet would be the most accurate and most informative way of obtaining the index of permeability.

Without the ability to directly measure the mass behind the sheet, it is still possible to estimate the index of permeability. This can be accomplished by instead measuring the mass contained in the sheet (m_{sh}) and subtracting it from the encountered neutral mass described above. In other words, the index would become:

$$\phi(t) = 1 - \frac{m_{\text{sh}}(t)}{m_{\text{avail}}(t)} \tag{10}$$

The mass contained in the sheet is from entrained ions and neutrals. If the sheet is found to have a ionization fraction of α_{sh}, then the term m_{sh} can be expressed as

$$m_{\text{sh}} = \frac{M^+ n^+_{\text{sh}}}{\alpha_{\text{sh}}} \tag{11}$$

where M^+ is the mass of the propellant ion. Also, using $n^e_{\text{sh}} = Zn^+_{\text{sh}}$, we can express the index of permeability in terms of the electron number density in the sheet:

$$\phi(t) = 1 - \frac{M^+ n^e_{\text{sh}}(t)}{Z\alpha_{\text{sh}} hd\rho \int_0^t v(t)dt} \tag{12}$$

Now we see that understanding the permeability of current sheets will require either a direct measurement of the trailing mass, or measurements of the current sheet's electron number density and velocity profile. Also, knowledge of the ionization fraction and presence of multiply ionized ions will allow a more accurate determination of the index.

We have described a method of measurement of electron number density in the sheet in the previous section. This laser interferometry method can obtain density information at a specific *point* in space for a range of time. More useful to us would be a system that can obtain density information in a two dimensional area of space at one specific point in time. Such a system is possible through fringe-counting interferometry methods and is one of the future measurement goals described below.

VI. REQUIRED MEASUREMENTS

This paper outlines some evidence for current sheet permeability in electromagnetic pulsed plasma thrusters. Several unanswered questions also are raised, such as what percentage of neutrals are ionized in the sheet, what speed the plasma wake is moving at, and what number of neutrals are permeating the sheet.

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[11] This would be more complicated if the mass is injected from the backplate, as in real thrusters

Answering these questions and others are the future goals of this study. Work will continue on making plasma measurements in an array of physical locations. We have seen above that measurements of plasma densities, neutral densities, sheet velocities, and other plasma parameters will be necessary to quantify an index of permeability and its dependencies on pressure, atomic mass of the propellant and canting angle.

Neutral density measurements would be beneficial for studying neutral plasma leakage through the sheet. Several diagnostic techniques have been mentioned for making these measurements, including two-color interferometry and fast dynamic pressure probes.

Electron density measurements in two dimensions at a specific time ("snapshot" interferometry) would be beneficial for analyzing the plasma densities in and behind the sheet. A fringe-counting Mach-Zehnder interferometer is planned for such measurements.

Magnetic field measurements have already begun as a method of obtaining velocity profiles of the sheet. These measurements are being taken with magnetic field probes developed by Markusic [5].

VII. CONCLUSIONS

Results are presented from theoretical and experimental analysis of electromagnetic pulsed plasma thrusters. The following conclusions can be drawn:

- The experimental velocity history of the current sheet in such devices is found to exceed the ideal snowplow velocity.
- A plasma wake is identified and seen to be of significant density compared to the current sheet. The wake also contains a larger fraction of ions at higher pressures.
- An approximate analysis shows that the sheet has entrained less than one third of the neutral particles encountered.
- An index of permeability is defined as the ratio of the trailing mass to the mass encountered by the sheet. It will be used as a means of studying the permeability phenomenon and its dependencies on various parameters.
- Measurements of plasma density, neutral density and current sheet speed are required to obtain the index of permeability and means of obtaining these measurements are discussed.

These conclusions all point to the existence of current sheet permeability which will negatively affect the performance of electromagnetic pulsed plasma thrusters. This study sets the stage for a more detailed and systematic study of the phenomenon.

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