Thruster concept for transverse acceleration by the beating electrostatic waves ponderomotive force

IEPC-2011-214

Presented at the 32^{nd} International Electric Propulsion Conference,
Wiesbaden, Germany
September 11–15, 2011

Benjamin Jorns* and Edgar Y. Choueiri†
Electric Propulsion and Plasma Dynamics Laboratory, Princeton University, Princeton, NJ, 08544

A thruster concept is presented for transversely accelerating a magnetized plasma by means of the ponderomotive force that arises when a plasma mode is absorbed while propagating across a uniform magnetic field. This concept offers a number of advantages for electric propulsion including electrodeless operation and variable specific impulse. A simple slab geometry is analyzed and a general expression for thrust is presented for this concept for arbitrary plasma modes. The normalized exhaust velocity as a function of wave frequency and input power is then derived for the specific case where the ponderomotive force is provided by the absorption of two, beating electrostatic waves. It is shown that this absorption process is particularly suited for the thruster concept and yields a specific impulse that can be tuned by changing the frequency of the exciting waves. A design is presented for an annular thruster that would implement this concept with beating electrostatic waves, and it is shown that this concept can produce significant exhaust velocities that are competitive with the Hall thruster for comparable power levels.

*Graduate Research Assistant
†Chief Scientist, EPPDyL, Professor, Applied Physics Group, AIAA Fellow

Nomenclature

$\mathbf{F}_{PM}$	Cold ponderomotive force
n_i	Ion density
$E_{1,2}$	Wave amplitude
$\omega_{1,2}$	Wave frequency
$k_{1,2}$	Wave number
ω_{pi}	Ion plasma frequency
n_i	Ion density
v_{ti}	Ion thermal velocity
v_ϕ	Wave phase velocity
Ω_s	Cyclotron frequency
q_s	Charge
$\mathbf{u}_s$	Fluid velocity
$\mathbf{j}$	Current density
$\mathbf{B}$	Magnetic field
m_s	Species mass
A_1	Thruster cross-sectional area
γ	Antenna coupling coefficient
η_A	Absorption efficiency
P_{RF}	RF input power
γ	Antenna coupling coefficient
α	Inverse length scale for wave absorption

I. Introduction

Plasma propulsion is an attractive candidate for many in-space applications due to the high exhaust velocities and moderate thrust densities that can be sustained with quasi-neutral plasma acceleration. One major limitation for the performance of many plasma propulsion systems, however, is the dependence of the acceleration mechanism on electrically conducting components that are exposed directly to the plasma. In the case of the magnetoplasmadynamic (MPD) thruster, for example, both significant anode and cathode erosion can result from the prodigious currents necessary to power the system.[1,2] Similarly, in Hall thrusters, erosion of the neutralizing cathode and thruster walls can limit both efficiency and lifetime of the thruster.[3] Since the advantages of these systems for deep-space missions depend in large part on long-term continuous operation, the limitation on thruster lifetime by electrode erosion can severely limit the range of applications of plasma propulsion.

As an alternative, a number of electrodeless thrusters have been proposed that employ oscillating electric fields coupled from antennae located externally to the plasma to achieve continuous acceleration.[4–7] One particularly promising thrust mechanism that can result from such oscillating fields is the ponderomotive (PM) force. This is a second-order effect that arises when there is a gradient in either the amplitude of the oscillating electric field in a plasma or the background magnetic field in the case of a magnetized plasma. The force is already widely used in fusion tokamak plasmas[8–11] to produce shear flows in the poloidal direction and has been proposed as a mechanism for current drive.[12,13] However, it also recently has been demonstrated that this this force can produce acceleration in a plasma propulsion concept.[6,7] In this cited configuration, the electrodeless plasma thruster, a combination of electron cyclotron waves launched from outside the plasma and an inhomogeneous magnetic field are employed to produce significant acceleration in the direction parallel to the magnetizing field.

While this concept has helped illustrate the efficacy of the PM force for achieving directed acceleration, it and similarly configured systems that produce acceleration parallel to the magnetic field are inherently limited in two ways. First, since the PM force in this case primarily accelerates electrons along the magnetic field, ambipolar coupling is necessary to produce thrust. This can significantly reduce the thruster's exhaust velocity. Second, the exit stage of the thruster depends on a diverging magnetic field such that the thruster efficiency may be limited by detachment losses characteristic of magnetic nozzle concepts.

In order to avoid these issues, we propose in this paper a new concept for an electrodeless thruster that relies on the PM force acting on a magnetized plasma. This concept has two distinguishing characteristics: 1) transverse acceleration (perpendicular to the magnetic field) of the plasma in order to avoid issues of detachment and 2) a body force on the plasma that accelerates primarily the ions as opposed to the lighter species. As we will discuss, these aims can be achieved in a plasma with a homogenous magnetic field by relying on the PM force produced by the natural damping of waves launched into the plasma.

This paper is organized in the following way. In Sec. II, we review the PM force in the cold-plasma limit. In Sec. III, we present a slab geometry with a homogenous magnetic field and illustrate how the PM force can produce transverse acceleration. For Sec. IV, we present a series of simplified fluid equations in order to provide a generalized estimate for the thrust of this concept. In Sec V, we discuss an attractive mechanism for producing the PM force in this concept–wave absorption by beating electrostatic waves (BEW)– and we numerically solve the normalized fluid equations for this case to estimate the final exhaust velocity as functions of frequency and input power of the BEW. Armed with these results, in Sec. VI, we present the conceptual design for an annular configuration that depends on the BEW PM force and estimate exhaust velocities for realistic input wave parameters and thruster geometry. We conclude with a discussion of the results and future theoretical and experimental investigations into this concept.

II. The cold ponderomotive force

The ponderomotive force density is a second-order effect that arises from the coherent, time-averaged interaction of the plasma motion induced by the wave with the perturbing induced wave fields. The self-consistent calculation of this force requires a full-kinetic treatment;[8,9] however, in order to illustrate the character of this force for the low temperature plasmas characteristic of many propulsion concepts, we employ in this work the analytical form of the PM force that is found in the cold-plasma limit. To this end, we consider a plasma subject to a wave field that has the form $\mathbf{E} = \sum_{m=1} \left[\mathbf{E}_{1m} e^{-i\omega_m t}\right]$ in a uniform magnetic field $\mathbf{B}$ where ω_m denotes the wave frequency of the m^{th} wave. For a cold-fluid approximation with multiple, non-coupled waves, the species (s) dependent form of the PM force is[14]

$$\mathbf{F}_{PM}^{(s)} = \sum_{m,s} \left[\frac{n_s}{4} \nabla \left(\mathbf{E}_{1m}^{*} \cdot \mathbf{p}_{1m}^{(s)} \right) - \frac{\omega_m}{q_s} \mathbf{B} \times \nabla \times \left(i n_s \mathbf{p}_{1m}^{*(s)} \times \mathbf{p}_{1m}^{s} \right) \right] \tag{1}$$

where we have summed over all of the wave frequencies and averaged over any slowly varying beat terms that may arise from $\omega_{m'} - \omega_m$. The term $\mathbf{p}_{1m}^{(s)} = \chi_m^{(s)} E_{1m}$ is the species-dependent polarization and χ_m is the cold plasma polarizability tensor $\chi_m^{(s)}$ given by[13]

$$\chi_m^{(s)} = -\frac{q_s^2}{m_s} \begin{pmatrix} \frac{1}{\omega_m^2 - \Omega_s^2} & i\frac{\Omega_s/\omega_m}{\omega_m^2 - \Omega_s^2} & 0 \\ -i\frac{\Omega_s/\omega_m}{\omega_m^2 - \Omega_s^2} & \frac{1}{\omega_m^2 - \Omega_s^2} & 0 \\ 0 & 0 & \frac{1}{\omega_m^2} \end{pmatrix}.$$

We can draw a parallel to single particle motion in order to understand physically the form of Eq. 1. In particular, the first term relates to the torque induced on an electric dipole from a non-uniform electric field while the second term corresponds to the induced torque on a magnetic dipole. In this case the dipole moments arise from a coherent interaction with the waves' oscillating field.

From Eq. 1, it can be demonstrated that the PM force acts in the direction of the gradient of the energy density determined by $|E|^2 = \sum |E_{1m}|^2$. The cold-plasma PM term therefore only has terms when the wave energy density is a changing function of position. For an illustration, we consider the case where $\mathbf{B} = B\hat{z}$ and $\mathbf{E} = E(x) e^{-i\omega t}\hat{x}$ such that the PM force density on ions exists exclusively in the direction of the gradient:

$$\mathbf{F}_{PM} = -\frac{n_i q_i^2}{4 m_i} \left(\frac{\omega^2 + \Omega_i^2}{\left(\omega^2 - \Omega_i^2\right)^2} \frac{\partial |E(x)|^2}{\partial x} \right) \hat{x}. \tag{2}$$

For multiple waves, if we ignore the slowly varying beat oscillations, then the static PM force densities of multiple waves are additive.

We reiterate here that the full PM force has a number of warm-plasma contributions that arise from kinetic power deposition in the plasma (the so-called hot Reynold's stress tensor contribution) as well as momentum transfer from direct absorption of the waves. Resolving the magnitude and direction of these forces requires a full kinetic calculation of the wave interaction with the plasma.[8,9] However, while these effects tend to me more pronounced in fusion (hot) plasmas for which they were originally derived, in the cold plasma limit of electric propulsion, we can assume that the cold PM force is the dominant force in second order quantities on the plasma. With this in mind, in this investigation, we consider exclusively the effect of the cold PM force for accelerating the species. The power deposited in the plasma will be accounted for separately by a simplified energy equation in Secs. IV and V.

We conclude this section by expressing the PM in the species fluid equations for a fully ionized plasma.[10] In this case we have reintroduced small thermal effects with a pressure term and ignored any viscous effects. These assumptions are valid for low temperature plasmas such that

$$\frac{\partial}{\partial t}\left[\rho_i \mathbf{u}_s\right] = q_s n_s \left(E_{int} + \mathbf{u}_s \times \mathbf{B}\right) - \nabla P_s + \mathbf{F}_{PM}^{(s)} + \mathbf{R}_s \tag{3}$$

where $\mathbf{u}_s$ denotes the species fluid velocity, E_{int} is a zeroth order electric field that arises from ambipolar effects, P_s is the pressure, $\mathbf{B}$ is the ambient magnetic field, and $\mathbf{R}_s$ is a drag term that can arise from electron-ion collisions.

III. Thruster concept

A. Geometry

In order to illustrate how the PM force can drive cross-field thrust, we show in Fig. 1 a simple slab geometry. The magnetic field is directed in the $\hat{z}$ direction with the PM force (on both species) in the $\hat{x}$ direction. The thruster is bounded by insulated walls in the $\hat{y}$ direction while it is open (but periodic) in the $\hat{z}$ direction. The plasma is pre-ionized in the channel.

For the moment, we postpone a discussion of the source of the PM force in this geometry and instead consider the dynamics of the system shown in Fig. 1. Since the PM force is independent of charge sign,

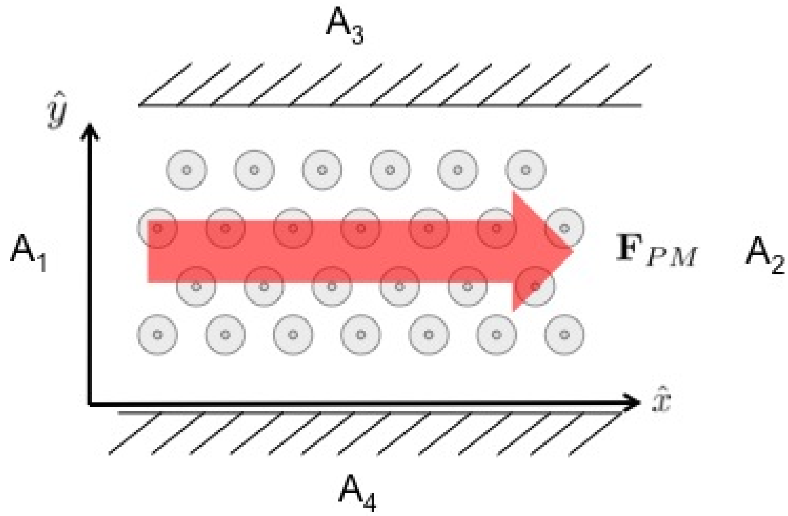


Figure 1. Geometry for ponderomotive force concept. The PM is directed in the $\hat{x}$ direction while the magnetic field is uniform in $\hat{z}$. The geometry is bounded by walls in the $\hat{y}$ direction.

we can see from Eq. 3 that the combination of the PM force and magnetic field will drive a current in the $\hat{y}$ direction. While for some configurations–primarily the Hall effect thruster where the electric field is the driving force– this current can be exploited to produce blowing force through a $\mathbf{j} \times \mathbf{B}$ interaction, in this geometry, the generated current works against any momentum imparted in the desired $\hat{x}$ direction. The purpose of the non-conducting walls in this case is to provide a means by which a charge imbalance can arise to reduce the current in the $\hat{y}$ direction through an ambipolar effect. This subsequently permits the PM force to accelerate the plasma in the direction in which it acts. This ambipolar assumption is a critical component for driving poloidal flows in tokamak geometry.[10]

Naturally a system without magnetic fields would not require a consideration of ambipolar effects to negate the Hall current, and it is possible that a PM force could be used in an unmagnetized plasma to accelerate a plasma with a properly tuned electric field gradient. However, we retain the magnetic field as shown in Fig. 1 in this concept for three reasons. First, despite the required ambipolar flux to the walls, the magnetic field will help confine the plasma. Second, the presence of a magnetic field facilitates the launching and efficient absorption of a number of magnetic plasma modes.[15] And third, as can be seen from Eq. 1 , for frequencies close to the cyclotron harmonic, the PM force density becomes quite large. This is an effect that does not have an equivalent for the unmagnetized ponderomotive force.

Given these advantages, we retain the magnetic field in our geometry and in the next section discuss two methods for generating a transverse PM force.

B. Generating PM force in the thruster concept

From the previous section, we saw that the cold PM depends both on the frequency of the waves, mass of the propelled species, and gradient of the wave intensity. While the mass of the propelled species is fixed and the frequency can be controlled externally, generating the gradient in the electric field is not a trivial task. This in part can be accomplished by shaping the antenna producing the oscillating electric field in the thruster geometry so as to reduce the field intensity along the thruster length. However, this poses a difficult engineering problem that requires a careful design of both antenna and impedance matching along the thruster length.

The more elegant solution we propose here is to launch a plasma wave in the $\hat{x}$ direction that is naturally damped by absorption mechanisms in the plasma (Fig. 2). This produces a naturally occurring gradient in the desired direction and only requires a single antenna to couple to propagating modes in the plasma. Moreover, this launching can be accomplished electrodelessly since some magnetized plasma modes, such as the electrostatic ion cyclotron wave (EICW) can be excited *inductively* from a source that is not in direct contact with the plasma.[16,17]

The additional advantage of this concept is that the gradient of the wave intensity depends directly on the absorption process, which in turn can be controlled by the frequency of the excited mode. For example, ion cyclotron damping depends on the proximity of the wave to the ion cyclotron frequency, Ω_i.[15] Therefore, for a wave with absorption that is primarily dominated by cyclotron damping, a steeper gradient in electric field can be achieved by tuning the frequency of the excited wave closer to Ω_i.

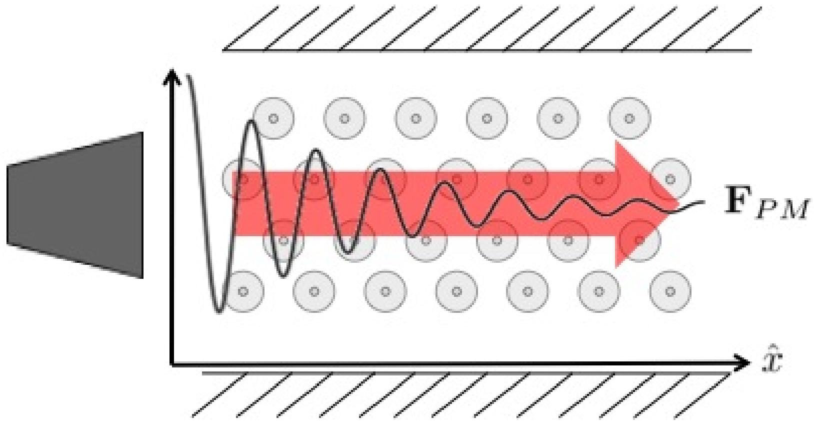


Figure 2. Propagating plasma modes are excited inductively from one end of the geometry by an antenna. The absorption of the wave spectrum by the plasma produces the necessary field gradient for a PM force.

For the above reasons of simplicity in implementation and controllability, producing the PM through a naturally absorbed plasma mode is an attractive alternative to trying to impose a gradient with a shaped antenna. This advantage is only temperated by the need to identify a plasma wave that will propagate perpendicularly to the magnetic field and be efficiently absorbed. We postpone a discussion of selecting a mode that will achieve this end for the moment, however, and in the next section without specifying a particular mode, we discuss the general scaling of the thruster with arbitrary wave parameters.

IV. Generalized Analytical Formulation

In order to examine acceleration in the thruster, we consider the single-fluid continuity and momentum equations:

$$\nabla \cdot \mathbf{j} = 0 \tag{4}$$
$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = \mathbf{j} \times \mathbf{B} + \mathbf{F}_{PM}$$

where we have assumed there is no ionization source in the plasma, ρ denotes the fluid mass density, u is the fluid velocity, $\mathbf{j}$ is the current density, and $\mathbf{F}_{PM} = \sum F_{PM}^{(s)} \hat{x}$, the total PM force density. Here, for simplicity, we have neglected pressure contributions.

In order to estimate the thrust, we integrate over the volume and examine the $\hat{x}$ direction. This yields

$$T_x = \int_V (j_y B)\, dV + \int_V \mathbf{F}_{PM} dV. \tag{5}$$

We further simplify this expression by assuming volume-integrated ambipolarity in the $\hat{y}$ direction such that $\int_V (j_y B)\, dV = 0$. This can be justified by volume integrating the continuity equation and assuming boundary conditions consistent with a quasineutral plume, non-conducting walls, and free streaming along the magnetic fields, i.e. $\int_{A1} j_x dA = \int_{A2} j_x dA = 0$ (quasi-neutral plume), $\int_{A3} j_y dA = \int_{A4} j_y dA = 0$ (non-conducting walls), and $\partial_z j_z = 0$ (free-streaming along magnetic fields). With these assumptions then, the momentum equation reduces to

$$T_x = \int_V F_{PM} dV. \tag{6}$$

When we substitute for F_{PM} from Eq. 1 and assume the force is uniform in the off-axis directions with constant density, we find

$$T_x = \sum_{s,m} \frac{n_s q_s^2}{4m_s} \left(\frac{\omega_m^2 + \Omega_s^2}{\left(\omega_m^2 - \Omega_s^2\right)^2} |E_m\left(0\right)|^2 \left[1 - \frac{|E_m\left(L\right)|^2}{|E_m\left(0\right)|^2} \right] \right) \tag{7}$$

where $x = L$ denotes the length of the thruster and we have dropped the subscripts on the electric field terms for convenience. Since we know that $|E_m(0)|^2$ scales with the intensity of each wave at the beginning of the acceleration zone, $x = 0$, if we assume that the energy flux density of each wave scales with this intensity (as is the case with electrostatic waves and some electromagnetic modes[15]), we can estimate that $\gamma P_{rf}^{(m)} = A_1 \alpha_{in(m)} |E_m(0)|^2$ where γ is an antenna coupling coefficient, $\alpha_{in(m)}$ is a wave dependent parameter, and $P_{rf}^{(m)}$ is the input wave power to each mode. Finally, let us denote the absorption efficiency as $\eta_{A(m)}(L, \omega_m) = \left(1 - |E_m(L)|^2 / |E_m(0)|^2\right)$ such that we find the following compact expression for the thrust

$$T_x = A_1 \gamma \sum_{s,m} \frac{n_s q_s^2}{4m_s} \left(\frac{\omega_m^2 + \Omega_s^2}{\left(\omega_m^2 - \Omega_s^2\right)^2} \frac{P_{rf}^{(m)}}{\alpha_{in(m)}} \eta_{A(m)}\left(L, \omega_m\right) \right). \tag{8}$$

This expression is quite general (subject to the cold PM assumption) and thus allows us to draw general conclusions about the scaling of thrust for this concept. In particular, not only does the thrust increase with input power P_{rf} as would intuitively be expected, but it also depends both on the launched wave through α_{in} as well as the absorption process and length-scale η_A. Specifically, from Eq. 8 it is evident that in order to maximize thrust for a given input power, it is necessary for 1) the absorption process to be as high as possible ($\eta_a \rightarrow 1$) with 2) a highly coupling coefficient $\gamma = 1$, and 3) a frequency approaching the cyclotron frequency of either species.

This controllability of thruster performance is an attractive feature of the thruster concept. We can gain more insight into the degree of this controllabillity by analyzing a specific wave combination and absorption process. This will permit estimates both for exhaust velocity u_{ex} and efficiency. To this end, in the next section, we consider a particularly attractive candidate for the waves to produce thrust through the PM force, beating electrostatic waves.

V. Thrust with Beating Electrostatic Waves

In order to motivate why BEW are particularly suited for this thruster concept, we consider from the previous two sections the characteristics that were desirable for the PM waves:

1. High absorption
2. Easily coupled to the plasma
3. Propagating transversely to the magnetic field
4. Frequency approaching the cyclotron frequency

The beating electrostatic wave process as outlined in Refs. 18–23 satisfies all of these criteria when electrostatic ion cyclotron waves (EICW) are employed. BEW absorption occurs when two electrostatic waves that propagate perpendicularly to the magnetic field in a plasma satisfy the so-called beating criterion: $\omega_2 - \omega_1 = \Omega_i$ where ω_1, ω_2 are the frequencies of the waves. This process has been shown to couple in a non-resonant way to a magnetized plasma, which can in turn lead to very high absorption of the EICWs as they propagate across the magnetic field lines.[22,23] This satisfies the first desirable conditions. Moreover, since the EICWs that produce the beating wave effect are electrostatic in nature and couple *nearly* perpendicularly to the magnetic field lines $k_{x1}, k_{x2} \ll k_z$, where k_{x1} and k_{x2} are the perpendicular wavenumbers of the excited modes, and k_z is the common parallel wave number, these modes can be excited efficiently, just as ion Bernstein modes are for tokamak plasmas, by current lines oriented parallel to the magnetic field lines.[16] These characteristics satisfy the second and third criteria. Finally, since EICWs exist in the ion cyclotron frequency range, the waves can be tuned close to the ion cyclotron frequency. This ensures that the waves can satisfy the final criterion. This ability to approach the cyclotron frequency is also an important consideration in that it implies the BEW PM force acts directly on the ions, the momentum carrying species of plasma.

BEW absorption through EICWs thus is an attractive candidate for producing thrust with the PM force. Furthermore, we can use the previously investigated characteristics of this process with EICWs to estimate both exhaust velocity and efficiency of the process. To this end, we again consider the momentum fluid equation, but in order to simplify the analysis further, we assume local ambipolarity in the $\hat{y}$ direction such that $j_y = 0$. While this assumption facilitates an analytical formulation for the exhaust velocity, its validity is compromised in the case when the charge imbalance at the walls is insufficient to ensure complete ambipolarity. In this event, the actual exhaust velocities in the thruster would be lower than those derived in the following investigation, though the results produced under the assumption of local ambipolarity still permit an estimate for the upperbound of exhaust velocity. With this in mind, we again neglect pressure contributions, and for further simplification, we exploit the fact that for EICWs, $\omega_1, \omega_2 \ll \Omega_e$ such that we retain only the ion contributions to the PM force. Under these assumptions, the momentum equation becomes

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\alpha_a \frac{\partial |E|^2}{\partial x} \hat{x} \tag{9}$$

where we have collapsed the coefficient for the ponderomotive force to $\alpha_a = \sum_{j=1,2} \frac{n_i q_i^2}{4m_i} \left(\frac{\omega_j^2 + \Omega_i^2}{\left(\omega_j^2 - \Omega_i^2\right)^2} \right)$ and consistent with BEW analysis, let the amplitudes of the two waves be equal. In this analysis, we also assume the mass density ρ is constant and reduce to the system to one dimension by allowing the waves to be uniform in the off axis directions and neglecting inertial contributions in the off-axis directions. This yields

$$\rho u_x \frac{u_x}{\partial x} = -\alpha_a \frac{\partial |E|^2}{\partial x} \hat{x}. \tag{10}$$

If we assume we know $|E(x)|^2$, we can integrate the above equation to find an expression for the exhaust velocity. For a zero initial fluid velocity $u_x(x=0) = 0$, we find

$$u_x(x) = \left(2 \frac{\alpha_a}{n_i m_i} |E(0)|^2 \left[1 - \frac{|E(x)|^2}{|E(0)|^2} \right] \right)^{1/2} \tag{11}$$

In order to find the spatial decay of the wave, we employ the result from Ref. 23 for BEW acting on an ion ensemble with a background temperature. In this case, the electric field produced by the waves is described by:

$$\mathbf{E} = E_0(x) \left[\cos(k_{x1} x - \omega_1 t) + \cos(k_{x2} xz - \omega_2 t) \right] \hat{z} \tag{12}$$

where we have neglected the parallel component of the wave number and the decay of the wave is encompassed by the spatial dependence of the wave amplitude $E_0(x)$. From Ref. 23, we see that the absorption of the BEW described by the above equation for a plasma with a fixed background ion temperature T_i yields the following result for the decay of the wave amplitude,

$$\left[1 - \frac{|E(x)|^2}{|E(0)|^2} \right] = 1 - e^{-\alpha x} \tag{13}$$

where the characteristic length scale is α^{-1}. This in turn is given by

$$\alpha = \left(\sum_{j=1,2} \frac{1}{k_{xi}} \frac{\omega_j}{\left(\omega_j^2 - \Omega_i^2\right)} \right)^{-1} \frac{m_i}{\Omega_{ci}^{1/3}} \frac{\pi}{8} \left(\frac{v_\phi^2}{k_{x1} T_i^{3/2}} \right)^{2/3} e^{-\frac{m_i v_\phi^2}{8T_i}}, \tag{14}$$

where $v_\phi = \omega_1 / k_{1x}$ is the phase velocity of the lower frequency wave. This result is valid provided the loss terms and escape velocity in the plasma are sufficiently high to prevent a significant change in the background temperature of the plasma. This assumption further supports our neglect of the pressure terms to the momentum equation. Physically, this result indicates that as the phase velocity of the waves approaches the thermal velocity of the background ions, the absorption process improves. Similarly, as the perpendicular wavenumber is decreased, the absorption process is also more pronounced.

The wave parameters described in Eq. 13 are constrained by the dispersion relation of the EICWs. This is discussed in detail in Ref. 23, but here we show the general dispersion relation for each wave[15]

$$\omega^2 = \Omega_i^2 + \frac{T_e}{m_i}k^2. \tag{15}$$

which is valid provided that $k_z \ll k_{1x}, k_{2x}$ and $v_{ti} < v_\phi < v_{te}$ where v_{ti} and v_{te} denote the ion and electron thermal velocities. This dispersion relation also allows us to relate the input RF power P_{rf}, to the initial wave intensity, i.e.

$$|E(0)|^2 = \gamma P_{rf}\left[A_1 \sum_{j=1,2}\left(\frac{\epsilon_0}{2k_j}\frac{\omega_j\omega_{pi}^2}{\left(\omega_j^2-\Omega_i^2\right)}\right)\right]^{-1}. \tag{16}$$

where ω_{pi} is the ion plasma frequency. Armed with this expression, we can find the exhaust velocity for a given set of plasma parameters as a function of position. In order to generalize our results, however, and provide more meaningful physical insight, we normalize our expression. To this end, we let

$$\tilde{u}_x = u_x/v_{ti} \qquad \tilde{x} = x\frac{\Omega_i}{v_{ti}} \qquad \tilde{\omega} = \omega/\Omega_i \qquad \tilde{k} = k\frac{v_{ti}}{\Omega_i} \qquad \varepsilon = \frac{\gamma P_{rf}}{A_1 n_i v_{ti}^3 m_i/\sqrt{8\pi}}. \tag{17}$$

where we have normalized all velocities by the thermal velocity $v_{ti} = \sqrt{T_i/m_i}$ and the lengths by the ion Larmor radius $r_L = v_{ti}/\Omega_i$. The last term is the ratio between the input wave power and the flux of thermal kinetic energy in the $\hat{x}$ direction given by $A_1 n_i\left(\frac{1}{2}T_i\bar{C}_s/4\right)$ where $\bar{C}_s = (8T_i/\pi m_i)^{1/2}$. This parameter is particularly important as we would not expect acceleration from the PM to exceed thermal effects until $\varepsilon \sim O(1)$.

With these normalizations and the above expressions, the normalized exhaust velocity from Eq. 18 is

$$\tilde{u}_x(x) = \left(\frac{\varepsilon}{(8\pi)^{1/2}}\frac{\sum_{j=1,2}\left(\frac{\tilde{\omega}_j^2+1}{\left(\tilde{\omega}_j^2-1\right)^2}\right)}{\sum_{m=1,2}\left(\frac{\tilde{\omega}_m}{\tilde{k}_m\left(\tilde{\omega}_m^2-1\right)}\right)}\left(1-e^{-\tilde{\alpha}\tilde{x}}\right)\right)^{1/2} \tag{18}$$

where the inverse of the characteristic normalized length is given by

$$\tilde{\alpha} = \frac{\pi}{8}\left(\sum_{j=1,2}\frac{1}{\tilde{k}_{xi}}\frac{\tilde{\omega}_j}{\left(\tilde{\omega}_j^2-1\right)}\right)^{-1}\left(\frac{\tilde{v}_\phi^2}{\tilde{k}_{x1}}\right)^{2/3} e^{-\frac{\tilde{v}_\phi^2}{8}} \tag{19}$$

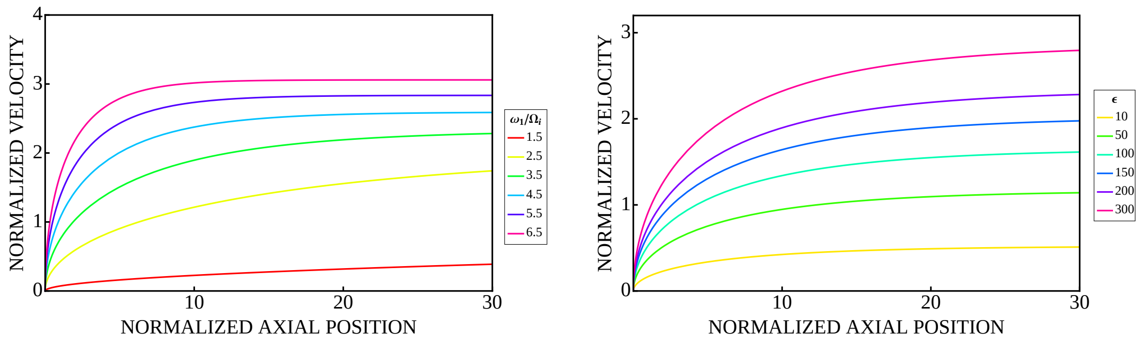
and the normalized dispersion relation is

$$\tilde{\omega} = 1 + \gamma_T\tilde{k} \tag{20}$$

where $\gamma_T = T_e/T_i > 1$ is the ratio of the electron and ion background temperatures.

At last, armed with these expressions, we can examine the exhaust velocity as a function of position and wave parameters. These are shown in Figs. 3 and 4 for a representative condition typical of low power thruster plasmas, $\gamma_T = 30$. In Fig. 3(a), we have plotted the normalized exhaust velocity as a function of normalized position for different values of BEW frequency $\tilde{\omega}_1$. It is evident from this plot that after a characteristic length, the exhaust velocity approaches an asymptotic value. This is a result of the absorption of the BEW such that at this characteristic length along the thruster, the wave amplitude and PM force approach zero. We also note in this plot that the length scale and magnitude of acceleration both increase with frequency. This is shown explicitly in Fig. 4(a) as well where we have plotted the asymptotic value of the exhaust velocity as a function of frequency. The reason for this trend is that the BEW absorption process improves as the phase velocity of the exciting waves approaches the thermal velocity of the ions. Since the phase velocity monotonically increases with frequency for EICW, the results shown in Fig. 4(a) are to be expected.

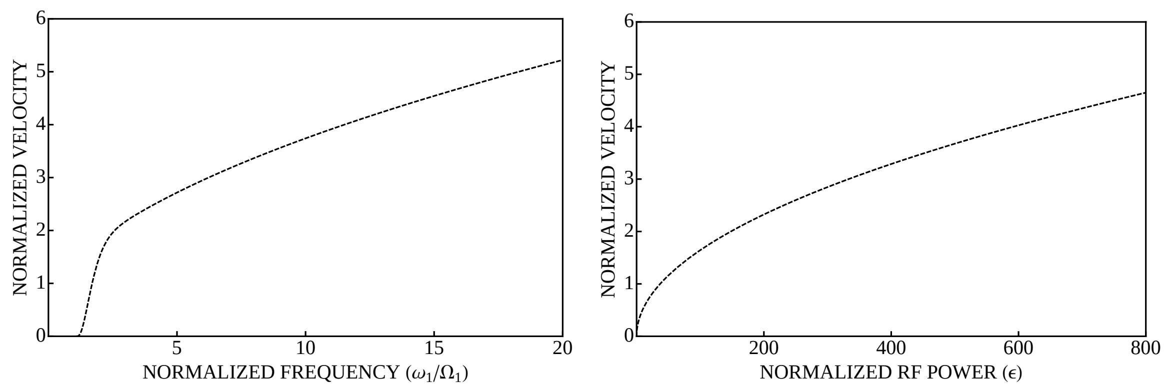
We also make the interesting note that even though the PM force diverges as the frequency approaches the cyclotron frequency $\tilde{\omega}_1 \to 1$, Fig. 4(a) indicates that the exhaust velocity in this limit is not only convergent



(a) $\varepsilon = 200$

(b) $\tilde{\omega}_1 = 3.5$

Figure 3. Plot of the trends in normalized exhaust velocity as a function of normalized position in the thruster. a) Each line represents a different set of BEW frequencies $\tilde{\omega}_1, \tilde{\omega}_2$ where $\tilde{\omega}_2 = \tilde{\omega}_1 + 1$. The normalized input power ε is constant. b) Each line represents a different normalized RF input power. The BEW frequencies are the same for all values shown.



(a) $\varepsilon = 200$

(b) $\tilde{\omega}_1 = 3.5$

Figure 4. The asymptotic values of the exhaust velocity as functions of normalized frequency and RF input amplitude. a) RF amplitude is constant. b) BEW frequencies are constant.

but zero. This stems from the fact that the EICW has a cutoff at the cyclotron harmonic, which dominates the divergent term in the PM force.

From Fig. 3(b), we can see the effect of increasing normalized RF power for a fixed set of BEW frequencies. In this case again, the exhaust velocity asymptotes with increasing length, and as Fig. 4(b) indicates, the magnitude of these asymptotic values increases monotonically with RF power. The reason for improved exhaust velocity with wave intensity in this case stems from the fact that the gradient of the wave intensity scales directly with $|E(0)|^2$ such that the cold PM is larger for greater input RF values.

Both of the trends outlined above provide means for controlling the exhaust velocity of the thruster. Either by increasing power to the waves or changing the wave frequency, a large range of exhaust velocities can be achieved. In addition to being electrodelesss then, producing thrust through the PM with BEW offers the considerable advantage of tunable specific impulse. In the next section, we outline in brief a thruster configuration that would employ the BEW, and we present some estimates for the exhaust velocity in this configuration.

VI. Annular geometry for PM thruster with waves

In order to exploit a closed geometry, for this concept we propose wrapping the slab geometry discussed above into an annular configuration shown in Fig. 5. In this case the magnetic fields are in the azimuthal direction and generated by a current carrying element along the centerline of the plasma. This element must be insulated from the plasma in order to ensure the no-current to the walls condition. The generated magnetic field in this case is assumed to be 500 G, which for the case of argon, translates to an angular cyclotron frequency of $\Omega_i = 120$ kHz.

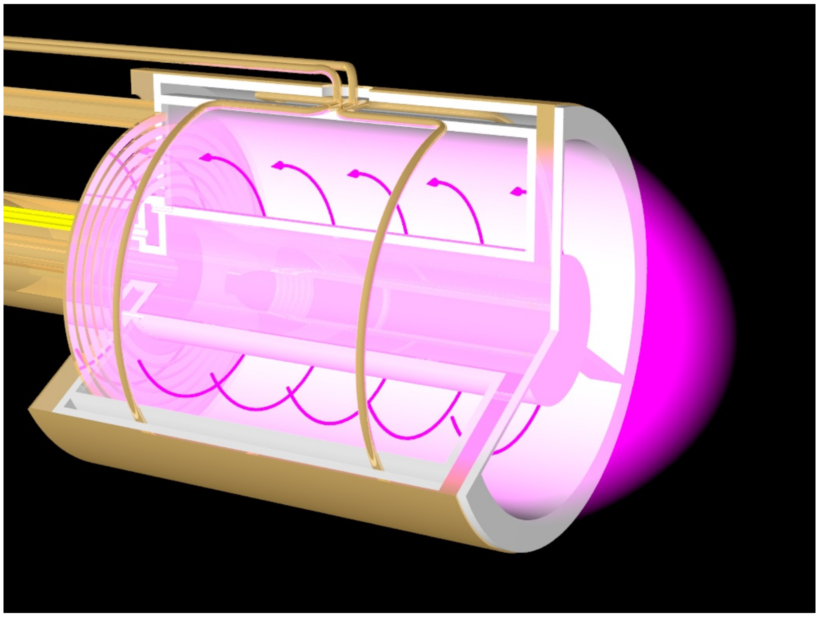


Figure 5. Annular configuration for the thruster concept that depends on the BEW ponderomotive force.

Argon gas is fed through the back-end of the thruster and ionized inductively by a saddle antenna placed concentrically around the volume of the thruster. The frequency of this ionization stage is typically > 13 MHz, such that the time scale of this oscillation is too fast to interfere with the low frequency EICW. This non-interference has been experimentally observed in laboratory plasmas as well.[24]

The two EICW necessary for this process are excited by a spiral antenna that is insulated from the plasma at the injection point of the gas. This loop antenna generates a net current in the azimuthal direction (along the magnetic field lines) that varies in time according to a signal that is the superposition of the two frequencies of the EICW. This current along the magnetic field lines allows direct coupling to the electrostatic modes in a way similar to the techniques employed in Bernstein wave launching in tokamaks.[16,25] The wavelength for this EICW mode is dictated by the geometry of the antenna which in this case is the circumference of the thruster. Provided the thruster diameter is sufficiently large, this will help ensure the near-perpendicularly propagating character of the EICWs.

The walls of the channel are chosen to be non-conducting in order to facilitate an ambipolar field that will cancel the Hall current as discussed before. This thruster concept is designed to operate in steady state with variable wave power and frequencies. In order to illustrate performance based on the previous, incompressible fluid discussion, we show in Fig. 6, the exhaust velocity as a function of axial position in the thruster for an input power of 1000 W and a thruster channel with an inner radius of 10 cm and outer radius of 20 cm. The following plasma parameters that are typical of an inductive plasma source are also used: $T_i = 0.1$ eV, $T_e = 3$ eV, $n_i = 10^{11}$ cm^{-3}. The antenna coupling is assumed to be unity, $\gamma = 1$.

As can be seen from Fig. 6, our one dimensional model suggests that exhaust velocities on the order of 20 km/s can be achieved through the PM force operating with BEW. For this low power level then, these results place this electrodeless concept in direct competition with the Hall thruster and moderate density plasma propulsion concepts.

VII. Discussion

From the above formulation, we see that this electrodeless concept for propulsion with PM force has significant potential for plasma propulsion. Our results have indicated through a one dimensional formulation that not only can this concept produce thrust, but the exhaust velocity can be scaled simply by changing the frequency of the exciting waves. This advantage complements well the inherent ability of this system to

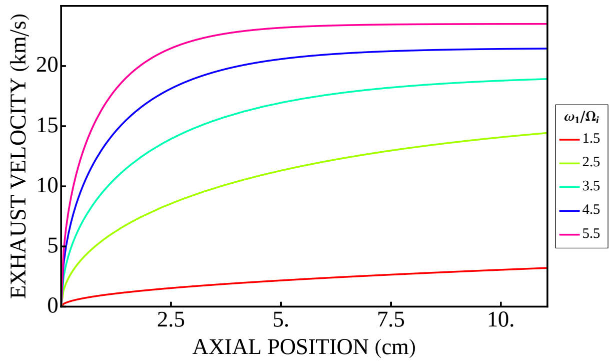


Figure 6. Exhaust velocity as a function of axial position in the thruster for argon with background ion temperature $T_i = 0.1$ eV, $\Omega_i = 120kHz$, $\gamma_T = 30$, and $P_{rf} = 1000$ W.

operate electrodelessly–therefore eliminating many erosion issues that are problematic for thrusters in the same power class. Our simple design for an annular manifestation of this thruster concept has also served to illustrate how this PM concept may be implemented and tested with relative ease.

Our analysis has been deliberately simplified in an effort to find easily evaluated expressions for the performance of the thruster. Some of these assumptions may modify in part the conclusions we have drawn above. In particular, we have assumed complete ambipolarity while neglecting any pressure effects that might be induced by the RF wavs. The former will tend to reduce our estimates for exhaust velocity while the latter, if a significiant pressure gradient is produced by the power deposition in the plasma, will amplify the exhaust velocity.

In addition, our model for the wave absorption taken from Ref. 23 fails to take into account the bulk velocity of the plasma. This is an important consideration as a net drift in the plasma may reduce the phase velocity of the excited wave in the reference frame of the plasma. This in turn could lead to improved coupling and absorption.

Finally, in our above results, we have assumed an incompressible flow. This has allowed for simple scaling relations for the exhaust velocity and thruster. However, for accurate esitmates of efficiency and thrust, the variation in density must be taken into account through the continuity equation.

All of these considerations can be addressed by employing a full simulation of the thruster as has been performed for a slab geometry of a tokamak.[8,9] This would allow for both self-consistency and violations of ambipolarity as well as a direct comparison with the simple scaling relations derived above.

VIII. Conclusions

In this paper, we have presented the design for a thruster concept that depends on the ponderomotive force produced by the natural absorption of waves that propagate across magnetic field lines. We have shown that this design offers significant advantages for electric propulsion both in terms of minimized erosion due to its electrodeless nature and an easily tuned exhaust velocity. We have presented a general analytical expression for thrust that is independent of the absorption process in the plasma and provided an analytical expression for the exhaust velocity when the PM force is provided by BEW absorption. The BEW process was shown to be particularly suited for this concept due both to the electrostatic, perpendicularly-propagating nature of the BEW, the ability of these waves to accelerate ions, and the high absorption for these modes that occurs in low temperature plasmas.

Finally, we presented a conceptual design for a thruster that would rely on the BEW process to produce PM and demonstrated that for physically reasonable plasma paramters and thruster dimensions, this concept could produce exhaust velocities on the order of 20 km/s for input powers of 1000 W. This result makes this concept competitive with equal power classes of thrusters such as the Hall thruster.

Acknowledgments

The authors would like to thank Robert Sorenson for help with a 3-D rendering of the annular configuration of the thruster and Greg Emsellem, Justin Little, and Matthew Feldman for several fruitful discussions about the merit of this concept. This material is based upon work supported by the National Science Foundation Graduate Research Fellowship under Grant No. 0646086 and the Program in Plasma Science Technology, Princeton Plasma Physics Laboratory.

References

[1]Polk, J., von Jaskowsky, W., Kelly, A., and Jahn, R., "Measurement of MPD Thruster Erosion Using Surface Layer Activation Authors," *Journal of Propulsion and Power*, Vol. 3, No. 1, Jan. 1987, pp. 33–38.

[2]Choueiri, E. and Ziemer, J., "Quasi-Steady Magnetoplasmadynamic Thruster Performance Database," *Journal of Propulsion and Power*, Vol. 17, No. 4, Oct. 2001.

[3]Goebel, D. and Katz, I., *Fundamentals of Electric Propulsion: Ion and Hall thrusters*, Wiley and Sons, Pasadena, CA, 2008.

[4]Toki, K., Shinohara, S., Tokai, T., et al., "On the Electrodeless MPD Thruster Using a Compact Helicon Plasma Source," *44th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, Hartford, CT, July 21-23, 2008*, No. AIAA-2008-4729, 2008.

[5]Satoh, S., Matsuoka, T., Fujino, T., and Funaki, I., "A Theoretical Analysis for Electrodeless Lissajous Acceleration of HELICON Plasmas," *42nd AIAA Plasmadynamics and Lasers Conference in conjunction with the 18th International Conference on MHD Energy Conversion (ICMHD), Honolulu, Hawaii, June 27-30, 2011*, No. AIAA-2011-4008, 2011.

[6]Emsellem, G., "Electrodeless Plasma Thruster Design," *41st AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, Tucson, Arizona, July 10-13, 2005*, No. AIAA-2005-3855, 2005.

[7]Emsellem, G., "Low Power Behavior of The High Power Electrodeless Plasma Thruster," *44th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, Hartford, CT, July 21-23, 2008*, No. AIAA-2008-5009, 2008.

[8]Jaeger, E. F., Berry, L. A., and Batchelor, D. B., "Full-wave calculation of sheared poloidal flow driven by high-harmonic ion Bernstein waves in tokamak plasmas," Vol. 7, No. 8, 2000, pp. 3319–3329.

[9]Myra, J. R. and D'Ippolito, D. A., "Toroidal formulation of nonlinear-rf-driven flows," Vol. 9, No. 9, 2002, pp. 3867–3873.

[10]Myra, J. R., Berry, L. A., D?Ippolito, D. A., and Jaeger, E. F., "Nonlinear fluxes and forces from radio-frequency waves with application to driven flows in tokamaks," Vol. 11, No. 5, 2004, pp. 1786–1798.

[11]Myra, J., D'Ippolito, D., Russell, D., Berry, L., Jaeger, E., and Carter, M., "Nonlinear ICRF-plasma interactions," *Nuclear Fusion*, Vol. 46, No. 7, 2006, pp. S455.

[12]Fisch, N. J., Rax, J. M., and Dodin, I. Y., "Current Drive in a Ponderomotive Potential with Sign Reversal," *Phys. Rev. Lett.*, Vol. 91, Nov 2003, pp. 205004.

[13]Gao, Z., Fisch, N. J., and Qin, H., "Nonlinear ponderomotive force by low frequency waves and nonresonant current drive," *Physics of Plasmas*, Vol. 13, No. 11, 2006, pp. 112307.

[14]Gao, Z., Fisch, N. J., Qin, H., and Myra, J. R., "Nonlinear nonresonant forces by radio-frequency waves in plasmas," *Physics of Plasmas*, Vol. 14, No. 8, 2007, pp. 084502.

[15]Stix, T. H., *Waves in Plasmas*, Springer, 1992.

[16]Cairns, R., *Radiofrequency heating of plasmas*, A. Hilger, 1991.

[17]Kline, J. L., Scime, E. E., Keiter, P. A., Balkey, M. M., and Boivin, R. F., "Ion heating in the HELIX helicon plasma source," *Physics of Plasmas*, Vol. 6, No. 12, Dec. 1999, pp. 4767–4772.

[18]Benisti, D., Ram, A. K., and Bers, A., "Ion dynamics in multiple electrostatic waves in a magnetized plasma. I. Coherent acceleration," *Physics of Plasmas*, Vol. 5, No. 9, 1998, pp. 3224–3232.

[19]Benisti, D., Ram, A. K., and Bers, A., "Ion dynamics in multiple electrostatic waves in a magnetized plasma. II. Enhancement of the acceleration," *Physics of Plasmas*, Vol. 5, No. 9, 1998, pp. 3233–3241.

[20]Strozzi, D. J., Ram, A. K., and Bers, A., "Coherent acceleration of magnetized ions by electrostatic waves with arbitrary wavenumbers," *Physics of Plasmas*, Vol. 10, No. 7, Jul 2003, pp. 2722–2731.

[21]Spektor, R. and Choueiri, E. Y., "Ion acceleration by beating electrostatic waves: Domain of allowed acceleration," *Physical Review E*, Vol. 69, No. 4, April 2004, pp. 046402.

[22]Jorns, B. and Choueiri, E. Y., "Ion Heating with Beating Electrostatic Waves," *Phys. Rev. Lett.*, Vol. 106, No. 8, Feb 2011, pp. 085002.

[23]Jorns, B. and Choueiri, E., "Efficiency of Plasma Heating with Beating Electrostatic Waves," *47th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, San Diego, California, July 31-3, 2011*, No. AIAA-2011-5894, 2011.

[24]Jorns, B. and Choueiri, E., "Experiment for Plasma Energization with Beating Electrostatic Waves," *46th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, Nashville, TN*, No. AIAA-2010-7107, 2009.

[25]Ono, M., "Ion Bernstein wave heating research," *Physics of Fluids B Plasma Physics*, Vol. 5, No. 2, 1993, pp. 241.