Design of an Experiment for Studying Ion Acceleration by Beating Waves

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The design and implementation of an experiment dedicated to the testing of recently derived criteria for ion acceleration by beating electrostatic (ES) waves are discussed. The non-resonant energization of the magnetized ions by the beating waves is potentially a highly-efficient process and thus of interest to propulsion. A cylindrical magnetized plasma source was constructed and uses either a helicon or inductive discharge designed to produce plasma parameters that allow testing the theory. The expected plasma parameters and the theoretical requirements led to an estimate of the frequency and intensity of the required electric field of the beating ES waves. The methodology for testing the effect is described along with the diagnostics needed to characterize the plasma and the effects of the injected waves.

I. INTRODUCTION

Radio Frequency (RF) plasma heating schemes are used widely in industry for material processing, scientific studies, and the electric propulsion. Typical RF heating mechanisms rely on resonance between waves and charged particles, as during ion cyclotron resonant heating (ICRH). In this type of heating mechanism a small fraction of magnetized ions, those in resonance with the waves, is able to gain energy from the waves and then distribute it to the other ions through collisional processes. An RF energization scheme that does not rely solely on the interaction with *resonant* particles, but can effect the entire distribution function, would theoretically be more efficient and thus of interest to a number of applications – especially propulsion. Such a non-resonant acceleration was shown by Benisti *et al.* [2] to result from nonlinear interaction between a magnetized ion and pairs of beating ES waves if the beating criterion, $\omega_1 - \omega_2 = n\omega_c$, is obeyed. Here ω_1, ω_2 are the frequencies of each wave, ω_c is the ion cyclotron frequency and n is a positive integer near 1.

Using numerical calculations along with second order perturbation analysis of a Hamiltonian formulation of the problem we were able to find the necessary and sufficient conditions for the effect to occur [5]. The existence of the effect remains to be systematically tested in a laboratory, although rocket-based observations in the ionosphere have been interpreted to provide evidence of the natural occurence of this effect [1].

Much as Skiff (Skiff, 1987) provided an experimental test of the theoretical predictions of Karney's (Karney, 1978) theory on the interaction between a *single* wave and a magnetized ion, we aim to provide experimental verification and testing of the new multiple wave theory.

This paper is organized as follows. In section II we provide a brief review of the theoretical framework and findings of our previous studies. In section III we propose a guideline for our experiments. In section IV we describe the design of the experimental apparatus that we developed to test the theory. Finally, we put our current work in perspective in section V and describe the direction of our future investigations.

II. SHORT REVIEW OF PREVIOUS THEORETICAL FINDINGS

A basic geometry of the idealized problem under consideration is shown in Fig. 1. We take the magnetic field to be constant in $\hat{z}$ direction, while electrostatic waves propagate in the $\hat{x}$ direction. For

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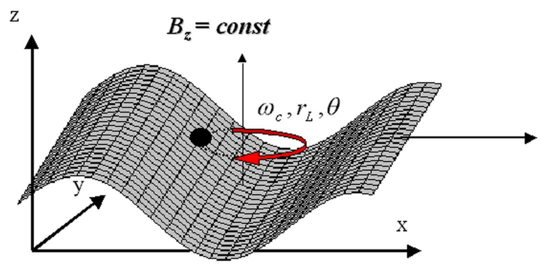

FIG. 1: Under collisionless assumption we consider a single ion in a constant magnetic field interacting with electrostatic waves propagating in x direction. For clarity only one wave is shown on the figure.

clarity, only one wave is shown on the figure. Any ion with a constant nonzero transverse velocity ($v_\perp$) will execute gyrating motion at a constant Larmor radius (r_L) with a constant ion cyclotron frequency (ω_c). It is important to remember that under the aforementioned conditions increase in the ion's Larmor radius corresponds directly to the increase in its velocity since $r_L = v_\perp/\omega_c$. The equation of motion of such system could be derived [2–5] to give:

$$\frac{d^2x}{dt^2} + \omega_c^2 x = \frac{q}{m}\sum_i E_i \sin(\mathrm{k}_i x - \omega_i t + \varphi_i), \quad (1)$$

where $\omega_c = q\mathrm{B}/m$ is the ion cyclotron frequency, E_i is the electric field, q is the electric charge, m is the mass, $\mathrm{k_i}$ is the wavenumber, and φ_i is the phase angle of each wave. It is impossible to obtain an exact analytical solution to this nonlinear equation. We therefore have to resort to numerical schemes or, alternatively, to a Hamiltonian formalism and then seek approximate analytical solutions using perturbation theory [6, 7]. The corresponding Hamiltonian for the system is

$$\bar{H} = \rho^2/2 + \sum_i \frac{\varepsilon_i}{\kappa_i}\cos(\kappa_i \rho \sin\theta - \nu_i \tau + \varphi_i). \quad (2)$$

In writing Eq.(2) we have used the fact that the system is periodic, and transformed the Hamiltonian into normalized action-angle coordinate system [6] where $\kappa_i = \mathrm{k}_i/\mathrm{k}_1$, $\nu_i = \omega_i/\omega_c$, $\tau = \omega_c t$, $\varepsilon_i = (\mathrm{k}_1 q E_i)/(m\omega_c^2)$, $\rho^2 = X^2 + \dot{X}^2$, and $X = \mathrm{k}_1 x$, $\dot{X} = dX/d\tau$, so that $X = \rho\sin\theta$, $\dot{X} = \rho\cos\theta$. The action-angle coordinate system is a special case of polar coordinates [7]. In our context θ corresponds to the cyclotron phase angle measured clockwise from the y-axis, as indicated on Fig. 1, while ρ is the normalized Larmor radius of the magnetized particle undergoing cyclotron motion in the $\hat{x}\hat{y}$ plane. Here again ρ is directly proportional to ion's velocity. As mentioned above, Benisti *et al.* [2] showed that to accelerate ions from arbitrary small initial velocities we need to have at least two waves such that

$$\nu_2 - \nu_1 = n \quad (3)$$

where $n =$ is a positive integer. In other words, this criterion, which we call BRB criterion (after Benusti, Ram, and Bers), requires that wave frequencies differ by an integer number times the ion cyclotron frequency.

Summary of fundamental features discovered by Benisti *et al.*

The two most important features of this acceleration mechanism are: 1) the acceleration is threshholdless in the sense that ions do not need to have *a priori* attained a certain velocity (resonance condition) to be able to be accelerated by the waves and 2) the acceleration is coherent until the ion is accelerated to the edge of the stochastic domain in phase space.

There are numerous other complex and rich features to this new acceleration mechanisms as we have discussed recently in some detail in [4]. We summarize here some of these features.

1. An ion, arbitrarily below the (stochastic) energization threshold required for single-wave acceleration, can pick up energy steadily and accelerate coherently.

2. The coherent and stochastic motion domains in phase space are connected unlike in the case of an interaction with a single wave.

3. Depending on the amplitude and frequency of the waves and the ion initial conditions, the ion can accelerate through the low energy boundary of the stochastic domain and be energized further stochastically.

4. The coherent acceleration time scales inversely with the square of the wave amplitudes.

These fundamental features have been discovered over the past four years mostly through parametric studies of the Hamiltonian. Many features remained largely unexplored analytically (until the first year of this project) and all features remain to be shown experimentally.

Insufficiency of the BRB acceleration criterion

During our numerical exploration of the problem [4] we discovered instances where an ion can become trapped in the coherent part of phase space and consequently cannot be accelerated by the beat waves despite the problem parameters that satisfy the BRB criterion (Eq. (3)).

This is displayed graphically in the plots of Fig 2 where we show the time history and Poincaré cross-sections of interactions at two different values of wave amplitude. As ε in that figure is raised from 49 to 50, (with all other conditions remaining the same) the ion is effectively locked out of stochastic phase space and its energy oscillates bounded by a maximum value of ρ. Since both interactions satisfy the BRB criterion (in this case, $\nu_2 - \nu_1 = 1$ with $\nu_1 = 24$ and $\nu_2 = 25$) it is clear that the **BRB criterion alone is *not* sufficient for thresholdless ion acceleration**.

A detailed analytical and numerical study [5] led us to the finding that the conditions under which an ion can become "trapped" can be defined in terms of the location (on the Poincaré section) of the critical points [7] of the motion. With that in mind, we used second-order perturbation theory and insight from numerical solutions to obtain analytical expressions for the location of these critical points. The analysis is quite involved mathematically [5] but, fortunately, leads to the following simple criteria that the Hamiltonian of the motion, $H(\rho;\theta)$, must satisfy in order for acceleration to occur (i.e. the particle not to be trapped),

$$H(\rho;\theta) < H_E \quad \text{or} \quad H(\rho;\theta) > H_H \tag{4}$$

where H_E and H_H are, respectively, the values of the Hamiltonian at the elliptic and hyperbolic critical points given by the following expressions

$$H_E = H(\rho \simeq \frac{\nu}{2} - \sqrt{\varepsilon}; \theta = \pi) \tag{5}$$

$$H_H = H(\rho \simeq \nu - \sqrt{\varepsilon}; \theta = \pi). \tag{6}$$

When these new criteria are combined with the BRB criterion in Eq. (3) we obtain a set of **necessary** *and* **sufficient** criteria for the acceleration to occur. The main goal of our initial experimental investigation is to investigate and validate this fundamental result.

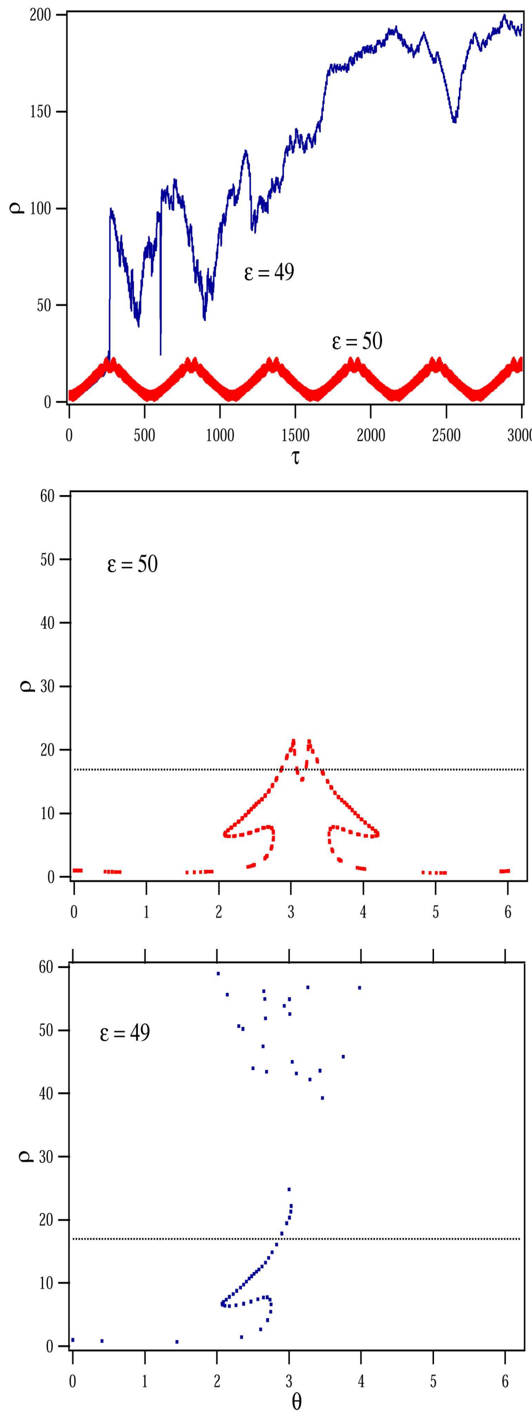

FIG. 2: Motion histories and Poincaré sections for $\varepsilon = 49$ and $\varepsilon = 50$.

III. TESTING METHODOLOGY, THE GOAL AND THE APPROACH

The goal of our experiments is to obtain for the first time an unambiguous and repeatable proof of the existence of this theoretically predicted thresholdless ion acceleration mechanism. By "thresholdless" here we mean, again, a mechanism that can accelerate ions of arbitrarily low initial velocity. We will seek to obtain this verification by conducting three sets of experiments. In the first the ion's velocity perpendicular to the magnetic field, $v_\perp$, will be monitored throughout the test section in the absence of energizing electrostatic waves. In the second set of experiments, $v_\perp$ (or rather its distribution function) will be monitored in the presence of a single electrostatic (ES) wave. While in the third set, the same experiment is repeated using two antennae that launch two ES waves whose frequencies and wavenumbers are chosen to satisfy the BRB criterion *and* the newly discovered criteria stated by Eq. (4). The ultimate goal of this task is to verify that all of the ions in the last set of the experiments are effected by the ES waves.

It is important to distinguish between two different regimes of ion acceleration during nonlinear wave-plasma interaction. When the wave amplitude is small, ions with small initial velocities will be accelerated coherently at first. Then, after reaching a certain threshold, their acceleration will become stochastic. The coherent acceleration is slow and takes few hundred to few thousand cyclotron orbits to complete.

When acceleration is stochastic, ions undergo rapid random changes in velocity. This allows ions to gain significant amounts of energy in a few cyclotron orbits. Typical change in the normalized velocity occurring within few cyclotron gyrations during regular acceleration is around $\Delta\rho_r \approx 0.05$ but during stochastic acceleration it increases to $\Delta\rho_s \approx 9.5$. Using values from Ref.[1] we see that velocity change during regular motion is on the order of 3 m/s while during stochastic acceleration the theory predicts that ions gain around 650 m/s during the same time interval.

Figure 5 shows that for a typical argon helicon discharge the ion cyclotron frequency is around 10^5 Hz while the coulomb collision frequency, the dominating collision type, is around 10^7 Hz. That means that there will be up to 100 collisions in a cyclotron cycle. It is unclear how collisions may effect stochastic ion acceleration, however there is evidence [10] to suggest that role of collisions is not always adverse.

Since the collision frequency varies linearly with density, we can reach the region where we could study coherent acceleration by decreasing density by two orders of magnitude. This could be achieved by lowering the fill pressure of the vacuum chamber and obtaining an inductively coupled discharge using the same set-up as is used to obtain helicon mode.

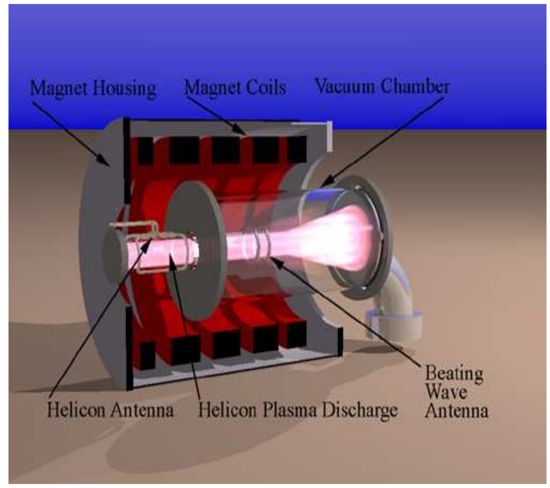

FIG. 3: Computer rendering of the experimental apparatus

FIG. 4: Beating Wave Experiment at EPPDyL.

IV. DESIGN OF EXPERIMENTAL APPARATUS

Vacuum Chamber

Rendering of the experimental apparatus is shown in Fig. 3. It consists of two pyrex cylinders placed in-

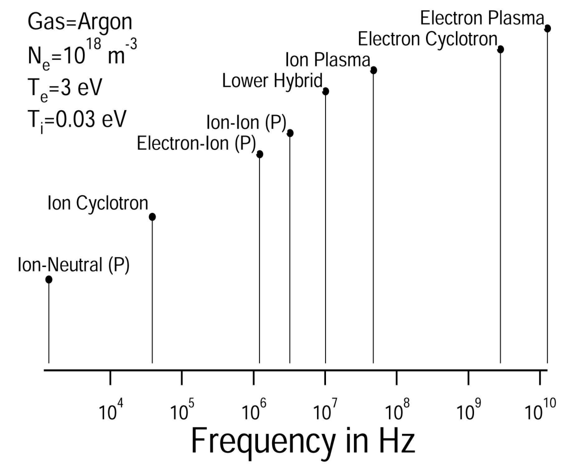

FIG. 5: Typical frequency values obtained in a helicon discharge.

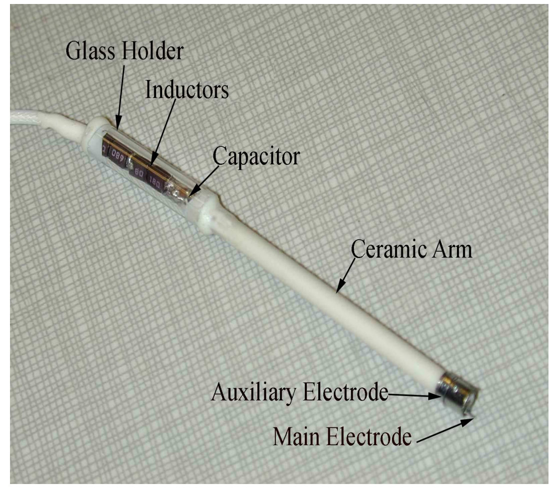

FIG. 6: Typical frequency values obtained in a helicon discharge.

side 0.1 Tesla magnet. The small cylinder is 6 cm in diameter and 25 cm in length while the large cylinder is 20 cm in diameter and 45 cm in length. The cylinders are connected by an aluminum plate with a 6 cm concentric whole at the center to allow free flow of gas between the two cylinders. A constant pressure of 0.1 to 30 mTorr is maintained by a gas feed (Ar or He) at the endplate of the large cylinder and by a 150 l/s turbo pump with a conductance controller as well as a roughing pump.

Once the discharge is ignited in the small cylinder, the plasma propagates along the magnetic field lines, which are parallel to the axis of the cylinders, into the large chamber where the beating waves are launched.

Plasma Source - Helicon Source

A Boswell type saddle antenna, used to produce helicon discharges, is placed around the small cylinder. The antenna is made of copper tubing to allow water cooling during operation. The helicon discharge is produced by supplying power to the antenna from an ENI 13.56 MHz 1.2 kW power supply through a tuner. The tuner consists of an L network made of two Jennings 1000 pF 3 kV variable vacuum capacitors. It was placed as close to the antenna as possible to maximize coupling.

In a typical helicon discharge the following plasma parameters are obtained: $N_e \approx 10^{12}$ cm^{-3}, $T_e \approx 3$ eV, and $T_i \approx 0.03$ eV. Figure 5 shows the range of some frequencies that are of interest in designing the experiment using a helicon source.

Inductive discharge was easily obtained with only a few watts of forward RF power to the antenna, however to ignite helicon discharge the power needed to be raised to at least 500 Watts. The inductively coupled discharge looks homogeneous and occupies the entire cross-section of the small cylinder. As RF power is raised above $\sim$500 Watts a helicon discharge can be obtained by properly adjusting pressure, magnetic field, and power. Special precautions must be taken in choosing backplate material for the antenna side. We used molybdenum to minimize the sputtering. When helicon discharge is obtained with argon, a bright blue column could be observed at the centerline. Since the characteristics of inductive (low ionization, collisions with neutrals dominate) and helicon (high ionization, coulomb collisions dominate) discharges are different we have two different regimes where we can investigate the beating wave-plasma interaction.

Required beating wave electric field and frequency

Ideally, we wish to operate over the following ranges of ν and ε that we've investigated with numerical simulations and analytical solutions:

$$10 < \nu < 100$$
$$5 < \varepsilon < 100$$

Whether this corresponds to a range of frequencies and amplitudes of waves that could be made to propagate in our particular plasma remains to be determined. This will be done after the characterization of plasma parameters using an RF compensated Langmuir [11] probe as shown in figure 6, and the calculation of the corresponding dispersion relation. Pending such results, it is informative to calculate the corresponding dimensional ranges of frequencies and electric field of the beating wave in our actual experiment.

The small tube diameter is 6 cm or roughly $2\pi \cdot 10^{-2}$ m. Plasma column will then be confined to that diameter. Let λ be an order of magnitude smaller: $\lambda = 2\pi \cdot 10^{-3}$ m. Thus the wavenumber is $2\pi/\lambda = 10^3\ m^{-1}$. Also for argon at B = 0.1 Tesla that leads to $\omega_c = 2.4 \cdot 10^5\ s^{-1}$. That leads to the following results:

$$\nu = \frac{\omega}{2.4 \cdot 10^5 \text{ s}^{-1}},$$
$$\varepsilon = \frac{E}{24 \text{ V/m}}.$$

Finally, the dimensional ranges are:

$$2.4 \cdot 10^5 < \omega < 24 \cdot 10^5 \text{ s}^{-1},$$
$$120 < E < 2400 \text{ V/m}.$$

V. CONCLUSIONS

We have thus presented the design and implementation of an experiment dedicated to testing previously derived criteria for ion acceleration by beating electrostatic (ES) waves. This dedicated experiment was designed, constructed and tested, using a helicon or inductive discharge source to generate plasma in which beating electrostatic waves are launched. We described the methodology for testing the effect along with the diagnostics needed to characterize the plasma and the effects of the injected waves. We have also estimated the frequencies and the magnitude of the electric field associated with the beating waves required to test the theory.

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