# Creation of Onset Voltage Hash by Anode Spots in Magnetoplasmadynamic Thrusters

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**Experimental results are presented which quantify the evolution with rising $J^2/\dot{m}$ (ratio of current squared to mass flow rate) of onset voltage fluctuations in magnetoplasmadynamic thrusters (MPDTs) operating with three anode materials, and an anode spot model is presented which provides a physical explanation for the properties of these fluctuations. Voltage signals taken in an MPDT operating below and above onset with anodes of copper, graphite, and lead are analyzed using the statistical measures of probability density and power spectrum. A model of voltage hash as the random superposition of many anode spotting events is used to generate voltage fluctuations with statistics similar to the observed data. The experimental fluctuation statistics evolve with rising $J^2/\dot{m}$ first away from Gaussian, and then back toward Gaussian, with the values of skewness and kurtosis peaking at $J^2/\dot{m} \sim 110$ kA$^2$-s/g; this behavior is the same for all three anode materials. Non-stationarity in the statistics is shown using high-speed video to be a result of unsteady anode evaporation. The statistics of modeled voltage hash are shown to be functions of the product of the frequency of anode spotting events and their duration, with the statistics becoming more Gaussian as this product grows. Comparison of experimental and model results suggests that, above $J^2/\dot{m} \sim 110$ kA$^2$-s/g, anode current conduction fragments into an increasing number of anode spots.**

## Nomenclature

| | |
|---|---|
| $J$ | MPDT total current |
| $\dot{m}$ | mass flow rate |
| $\Gamma$ | anode mass flux |
| $p_{sat}$ | saturation vapor pressure |
| $M$ | atomic mass |
| $k$ | Boltzmann constant |
| $T$ | temperature |
| $f$ | frequency |
| $d$ | anode mark size |
| $V$ | voltage |
| $I_e$ | total spot current |
| $C$ | sheath capacitance |
| $R$ | spot resistance |
| $n$ | spot event occurrence rate |
| $\tau$ | spot event rise time |

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# I. Introduction

The onset problem in magnetoplasmadynamic thrusters (MPDTs) has been a subject of study for several decades. In that time, many observations regarding the critical current above which onset phenomena are likely to appear have been made: in particular, that the critical value of the parameter $J^2/\dot{m}$ can be changed by suitable alterations of the thruster geometry[5,18,21] or the propellant type.[18] Many theories have been put forth to try to explain the critical value of $J^2/\dot{m}$, but the most well-supported theory is that of anode starvation, which states that the thruster will enter onset when the total current exceeds the sheath-limited current-carrying capability of the near-anode plasma.[2,8,14]

What has received less attention is the behavior of the thruster when onset has been exceeded—that is, the behavior of the fluctuating voltage and the damage caused on the anode that are both hallmarks of a thruster operating above onset. As to the latter, Vainberg et. al.,[28] Hugel,[14,15] and Diamant[8] have all observed that anode damage occurs in discrete spots. Rudolph[22] and Ho[13] have shown that evaporation of anode material occurs at $J^2/\dot{m}$ similar to that at which voltage fluctuations begin. The voltage fluctuations themselves have been correlated with fluctuations in other thruster parameters, such as the optical emission,[17] electric and magnetic fields,[3] and plasma density.[9] In our previous work,[27] we have shown a correlation between voltage hash and the emission of light from anode material in the discharge—though it remained unclear what the relationship between the two actually is.

It is our desire to understand the origin of the voltage fluctuations ("hash"), so that this knowledge can be applied to the development of ways to circumvent or at least delay in $J^2/\dot{m}$ the appearance of the hash and perhaps also the anode damage. In this paper, we undertake to present an experimental study of the voltage hash, focused on identifying quantifiable descriptive measures for the evolution of the hash with $J^2/\dot{m}$ and with changing anode material. We calculate the relevant statistics that describe the hash, show that the observed anode damage is consistent with our voltage hash results, and then present a model useful for generating voltage hash with the same statistics as those we see in our data. Throughout, we will make reference to anode spots—that is, small regions of enhanced current conduction at the anode—as a basis for explaining aspects of voltage hash and anode damage.

We begin in Sec. II by describing the specific MPDT system used in this paper. Section III describes the voltage hash in a statistical sense, using the probability distribution and power spectra of the signals at various $J^2/\dot{m}$. Also in Sec. III, we explain the origin of non-stationary statistics using high-speed video to capture the effect of anode vapor jets on the voltage hash. In Sec. IV, we present some observations of the anode damage caused by onset, and describe how these are consistent with the observations of voltage hash. Section V describes a model that we use to generate signals that accurately represent voltage hash, with the whole range of observed statistics, and use that model to gain some insight into the behavior of anode spots with changing $J^2/\dot{m}$.

# II. The Princeton Benchmark MPDT

## A. FSBT System

The Princeton Full-Scale Benchmark MPDT (FSBT) system used in this study has been used since 1983 in extensive studies of thruster performance,[6,7] electrode erosion,[20] and plasma properties. The dimensions of the thruster are shown in Fig. 1. For the experiments in this paper, the thruster was fed with argon propellant at rates of 3 and 6 g/s, equally split between the cathode annulus and the outer ring of 12 holes shown in the figure (we will, however, use the well-known MPDT scaling parameter $J^2/\dot{m}$ to present all data, so that the mass flow rate will not appear explicitly). The cathode is tungsten, the anode made of various materials (as we shall discuss), and the insulating surfaces are Pyrex and boron nitride.

The FSBT is operated in quasi-steady fashion, using a pulse of current supplied by a pulse-forming network (PFN) with a 12.9 mF total capacitance and 14.9 $\mu$H total inductance. This PFN supplies 1 ms flat-topped pulses of current up to 25 kA. A stainless steel ballast resistor matches the impedance of the thruster to that of the bank, preventing the current from ringing during the pulse. An air-fed gas switch holds off the full PFN voltage from dropping across the thruster until triggered.

The voltage across the thruster is measured as physically close as possible to the thruster electrodes, to eliminate the influence of power supply noise.[26] The signal is digitally sampled by a recording oscilloscope at a rate of 40 ns per point, for a Nyquist frequency of 12.5 MHz. Before digitization, the signal is filtered by a six-pole low-pass filter with a cutoff frequency of 5 MHz.

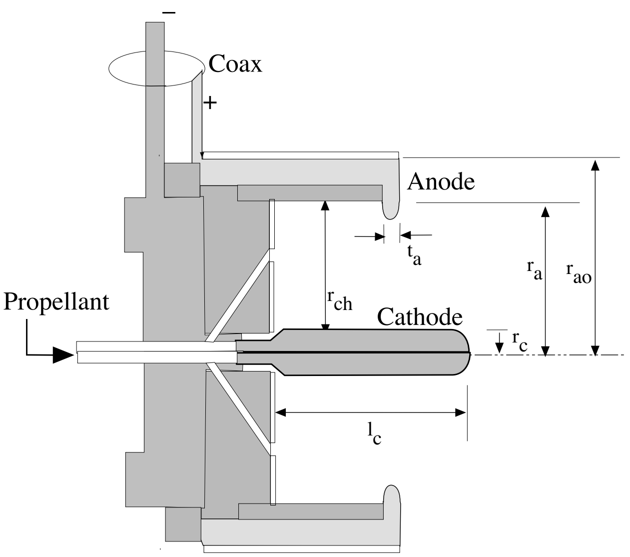

**Figure 1. The Princeton Benchmark MPDT.** $r_c = 0.95$ **cm,** $r_a = 5.1$ **cm,** $r_{ao} = 9.3$ **cm,** $r_{ch} = 6.4$ **cm,** $t_a = 0.95$ **cm, and** $l_c = 10$ **cm.**

In one section, we present frames of a high-speed video taken of the thruster during an above-onset firing. The videos from which the presented frames were selected were taken with a Photron Ultima APX high-speed video camera at 50,000 fps. The light entering the camera was bandpass filtered at 630 nm to attenuate propellant (argon) emission and allow anode (copper) emission from the arc. The camera was triggered by an auxiliary output of the recording oscilloscope, so that the videos were synchronized with the voltage data acquired.

### B. FSBT Anode Materials

The anodes used in the FSBT for this study were made of four materials: copper, aluminum, lead, and graphite. Copper and aluminum have been the most recently used to study the near-anode plasma parameters in the FSBT,[9] but the thermal properties of these two materials are quite similar with respect to the full range available in metals. Lead, for example, melts at 601 K, while graphite does not melt at all.

As regards the interaction of the anode with the discharge, the most relevant measure of the difference between materials is the erosion rate. The erosion rate into vacuum of any material is a function of the temperature—derivable from kinetic theory—and is given in SI units by

$$\Gamma = p_{sat}\sqrt{\frac{M}{2\pi kT}}, \tag{1}$$

where $p_{sat}$ is the equilibrium vapor pressure. Empirical curves are available for $p_{sat}$ for all of the above four materials;[1,11] using these (with the three metals in the liquid phase, and graphite solid), we calculate the erosion rate $\Gamma$ for each as plotted in Fig. 2.

It is clear that copper and aluminum have very similar erosion rates over the range of temperatures that the anode experiences in the FSBT, in comparison to the difference between lead and graphite. An FSBT operating with a lead anode will have significantly more interaction between the arc and the anode than one operating with a graphite anode. The wide disparity in erosion rates will highlight the influence of erosion on onset voltage hash, as we will discuss.

## III. Properties of Onset Voltage Hash

The 1 ms current pulse described in the last section is provided by a PFN of sufficient inductance that the current does not change appreciably if the thruster impedance varies during the course of a firing. The

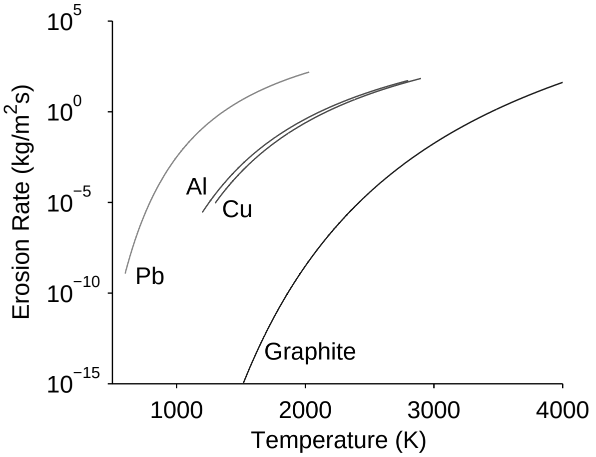

**Figure 2. The erosion rate of four anode materials as a function of temperature.**

hallmark of the onset problem is that, while at low currents the thruster impedance is constant (about 10 m$\Omega$) during a firing, at high currents the impedance changes rapidly—which, due to the stiff current source, causes the voltage to do the same. In the FSBT, this transition occurs at $J^2/\dot{m} \sim 60$ kA$^2$-s/g. The magnitude of this fluctuation grows rapidly beginning at a higher value of $J^2/\dot{m} \sim 80$–100 kA$^2$-s/g. Examples of the voltage hash after each of these transitions are shown in Fig. 3.

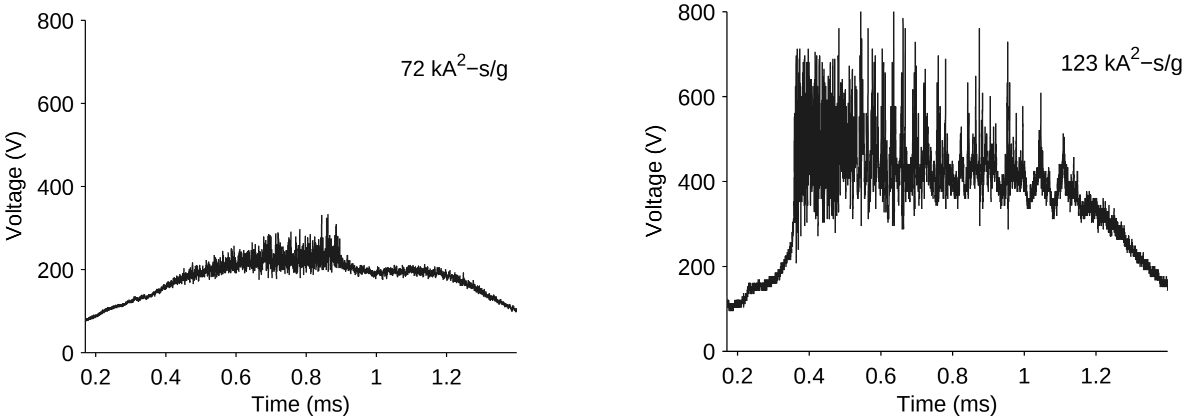

**Figure 3. Example voltage hash traces, at the $J^2/\dot{m}$ levels indicated.**

As discussed in the last section, we have obtained voltage hash using anodes of several materials. In what follows, we will discuss the hash obtained with copper, lead, and graphite anodes.

## A. Statistics of Voltage Hash

The voltage hash that we have measured in this study is a random fluctuation. “Random” is a technical descriptor that indicates that no deterministic (that is, transient or periodic) function can be used to describe the fluctuation of the voltage during periods of hash. The distinction between periodic and random data can be made based upon whether peaks appear in its power spectrum: as we will see, this is not the case for our voltage hash. At the same time, it is clear that our voltage hash is not a transient function of time.

The analysis of random signals is more complex than the corresponding analysis of deterministic signals, as discussed in the classic work of Bendat and Piersol.[4] Whereas a deterministic signal is described by an equation with parameters that can be determined by analysis of the signal—and presumably varied by alterations to the experiment—random data must be analyzed in a statistical sense. Statistics generally calculated in the process are the mean, the spectral density, and the probability density of the signal (which is often Gaussian about the mean, but can differ significantly from this baseline).

We will use this analysis to describe our voltage hash. However, applying these measures to the analysis of voltage hash presents unique challenges because each signal is nonstationary, and the statistics are therefore not constant over the duration of each signal. For example, perhaps the most obvious feature of the voltage traces of Fig. 3 is that the voltage hash is not of constant amplitude over the course of the firing. This is not the normal behavior of hash amplitude during a firing—in fact, there is no "normal" behavior that we can ascribe to the time variation of the hash amplitude. In a series of firings under identical experimental conditions, the hash may last for the entire quasi-steady portion of the firing, or may begin and end one or more times in bursts throughout the firing. As we will discuss, this behavior has to do with the release of vaporized anode material into the discharge. We cannot, however, control this behavior by any standard modifications to the MPDT—its circuit elements, anode condition, or propellant. As a result of this variability, the statistics that we calculate for the voltage hash exhibit a degree of scatter.

Our analysis proceeds as follows. For each voltage hash trace as in Fig. 3, we take a portion of the trace corresponding to the time of the quasi-steady current (which lies between 0.4 and 1.2 ms) and calculate the probability distribution of the signal during a 0.3 ms portion of this time (doing so is important to avoid startup and shutdown transients, which can be longer in the voltage signal than they are in the current, and can cause significant scatter in the statistics). For better spectral resolution, we calculate the power spectra using the entire quasi-steady period. The statistics and spectrum of each signal are then averaged over several identical firings. Error bars on all such quantities represent the scatter in the data. The exception to this rule is data taken with a lead anode, which sustains significant damage on each firing. Only a single firing was taken with lead at each $J^2/\dot{m}$, so that these data lack error bars and the power spectra have greater spectral noise.

### B. Results

The mean, standard deviation, skewness, and kurtosis (the first four standardized moments) of the voltage hash obtained with copper, lead, and graphite anodes are shown as functions of $J^2/\dot{m}$ in Fig. 4. The signal mean is is the average voltage over the course of a firing. The standard deviation is a quantitative measure of the amplitude of the hash: for a Gaussian distribution, 95% of the voltage signal would be contained within two standard deviations of the mean. Using this measure, rather than a peak amplitude, guards against overestimating the hash magnitude in cases where an outlying fluctuation is much larger than the typical fluctuations.

The skewness is the measure of any "long tails" on either the positive or negative side of the distribution average. A positive skewness indicates that a signal spends more time above the mean value than a Gaussian signal would. (A Gaussian distribution, which has no long tails, has a skewness of zero.) The kurtosis of the signal is the measure of how "peaked", or biased toward small values, the signal is. A signal which spends much time near the mean, with short-duration, large excursions away will have a positive kurtosis. (Again, a Gaussian distribution has a kurtosis of zero, in our definition; other definitions assign a Gaussian a kurtosis of 3.)

The plot of the mean values is a classic voltage-current characteristic of the self-field FSBT.[7] We will not dwell on this, except to mention that at the highest currents, a lead anode appears to operate at a lower voltage than the others. Because lead provides a more copious supply of evaporated material to the discharge than the other anodes, the plasma density near a lead anode will likely be larger than that near an anode of another material, and lower voltages would be necessary to drive equivalent current through this more-conductive plasma. This may be seen as equivalent to having a higher effective $\dot{m}$.

The standard deviation of the voltage signals follows the same trend over the range of current for all three anodes. It is somewhat surprising that the magnitude of the hash, or the $J^2/\dot{m}$ level at which it grows significantly, is apparently insensitive to very different anode materials—anode evaporation, after all, has been associated with voltage hash many times in the past and it is reasonable to think that the one affects the other. On the other hand, it is also possible that the voltage hash arises independently of the anode damage, which is itself a passive thermal response of the anode to the mechanism behind the hash. We will explore one such possible mechanism in Sec. V.

The skewness and kurtosis of the hash also follow similar trends among the materials. Each hovers around zero over most of the current range, before rising to large positive values at the $J^2/\dot{m}$ at which the standard deviation begins rising significantly. This happens because, as the hash grows between the magnitude levels in the two panels of Fig. 3, the fluctuations first resemble infrequent, large positive excursions away from

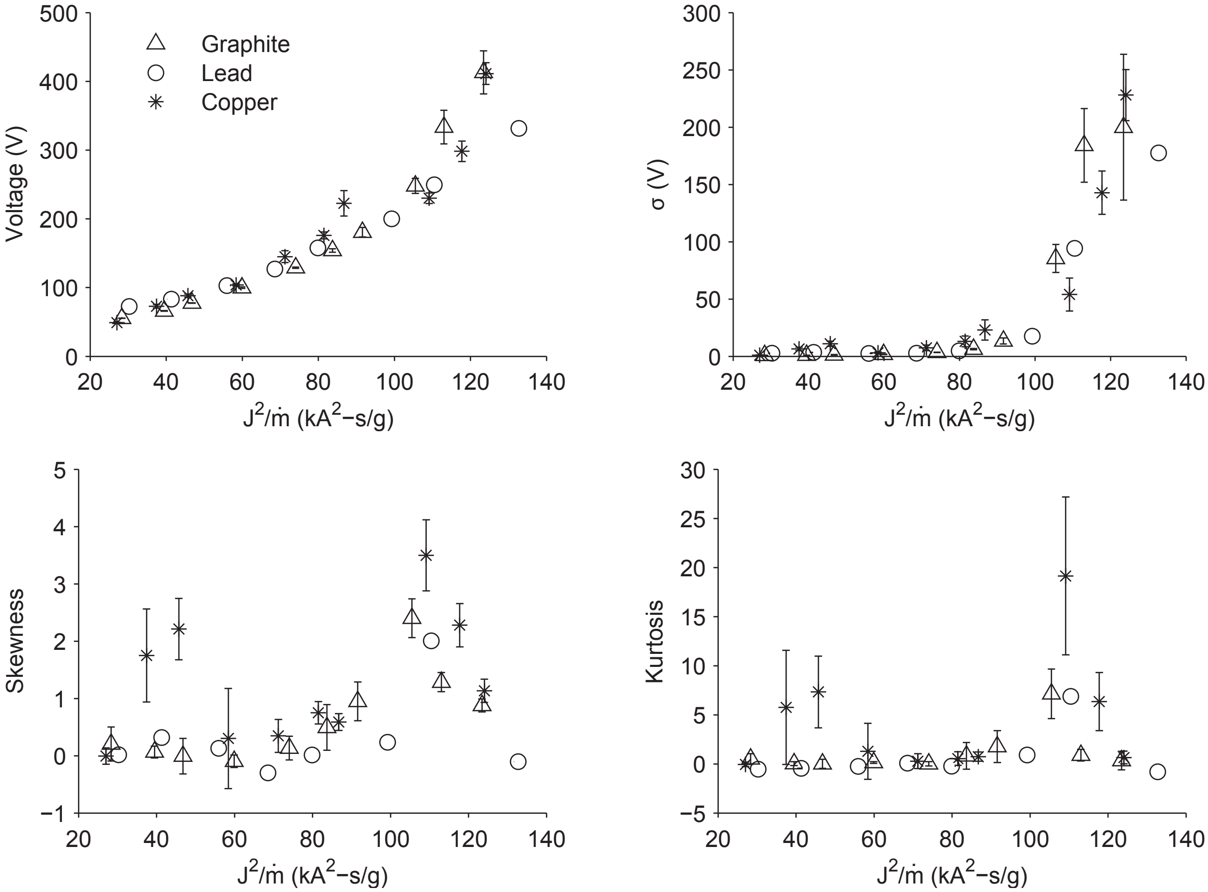

**Figure 4. Voltage statistics for three anode materials.**

the mean. As $J^2/\dot{m}$ rises further, these excursions become more frequent and begin to overlap, bringing the skewness and kurtosis statistics back toward their Gaussian levels. That the excursions occur above the mean gives rise to the positive skewness; the positive kurtosis occurs because the excursions rise and fall quickly. This behavior indicates that the random fluctuations in the voltage arise from ascending spikes, which relax quickly to small values. Again, we will discuss one possible origin for this behavior in Sec. V.

The skewness and kurtosis for copper have a brief excursion away from zero at low $J^2/\dot{m}$ values. This is because hash with a copper anode, unlike that with lead and graphite, does not arise smoothly out of low-current quiescent operation, but first occurs in short bursts during a single firing. This is likely due to the influence of anode erosion, which we will see in Sec. III.C. Refractory graphite may not erode enough to affect the hash in the same way, while lead may erode more continuously, so that the effect on the hash does not change during the firing. Copper, which as we will see erodes in bursts, lies in the middle of those two extremes.

The similarity of the statistics for the three anodes is paralleled in the similarity of the power spectra for the same three anodes, an example of which we give in Fig. 5. All three spectra display the same $1/f^\beta$ characteristic fall, with $1 < \beta < 2$. (Periodic content at 1.3 MHz is noise associated with the power supply[26]).

The $1/f^\beta$ characteristic fall in the spectra is characteristic of a Brownian motion ($\beta = 2$ is true Brownian motion, while $\beta < 2$ is a fractional Brownian motion), and is revealing with regard to the mechanism underlying the hash.[23] Whereas a white noise process, which has a flat spectrum, is generated by choosing a random value for each point in the signal, a Brownian motion (specifically, for $\beta = 2$) is generated by choosing a random value for the *slope* between each two points in the signal—and hence the Brownian motion is the integration of white noise. The spectrum of an integrated white noise signal falls off with a slope $\beta = 2$. As we consider the physical process behind the voltage hash, therefore, we must keep in mind that the underlying randomness in the signal is not in the sampled voltage values themselves, but in the voltage change between consecutive samples. We will use this insight in Sec. V to develop a random model for the hash.

### C. Origin of Non-Stationary Statistics

We have already noted that the statistics of the voltage hash are non-stationary—that is, they change over the course of a single quasi-steady firing. This non-stationarity is a result of anode evaporation, which, if severe enough, can suppress the hash. This effect of anode evaporation can be captured using high-speed video.

A representative example of our video footage appears in Fig. 6. This shows, at top, a portion of a voltage trace for a thruster firing with a copper anode at $J^2/\dot{m} = 60$ kA$^2$-s/g, labeled on the abscissa with numbers corresponding to the numbered frames of the high-speed video shown at the bottom of the figure. For brevity, we show just two such frames; even from these, the conclusion is obvious.

The two frames, each of which captures 20 $\mu$s, have been inverted and contrast-enhanced, so that dark regions correspond to copper emission, and white regions correspond to darkness. We have superimposed dotted lines to represent the internal edge of the anode, and another to represent the cathode, which is the white area surrounded by black pixels in both frames.

The pattern revealed in our videos is clear: the emission of anode vapor in jets that arrive at the cathode tends to suppress the voltage hash, which drops to a small fraction of its magnitude whenever the eroded copper forms a bridge between the anode and cathode, as in frame 10. On the other hand, the periods in which large-amplitude voltage hash exists correspond to the periods in which eroded anode material does not form such a bridge. This is consistent with the observation from vacuum interrupter literature that voltage hash in vacuum arcs is suppressed when the evaporated material in an anode jet bridges the gap between anode and cathode.[12]

## IV. Properties of Onset Anode Damage

We have so far discussed the voltage hash, noting that it is a random fluctuation with statistics that evolve with $J^2/\dot{m}$ but which are not greatly altered by the choice of anode material. This finding argues that the voltage hash exists to a great extent independently of the damage to the anode, and that the anode is therefore a passive responder to the mechanism underlying the hash (this, with the exception that sufficient erosion can suppress the hash, as we saw in the last section).

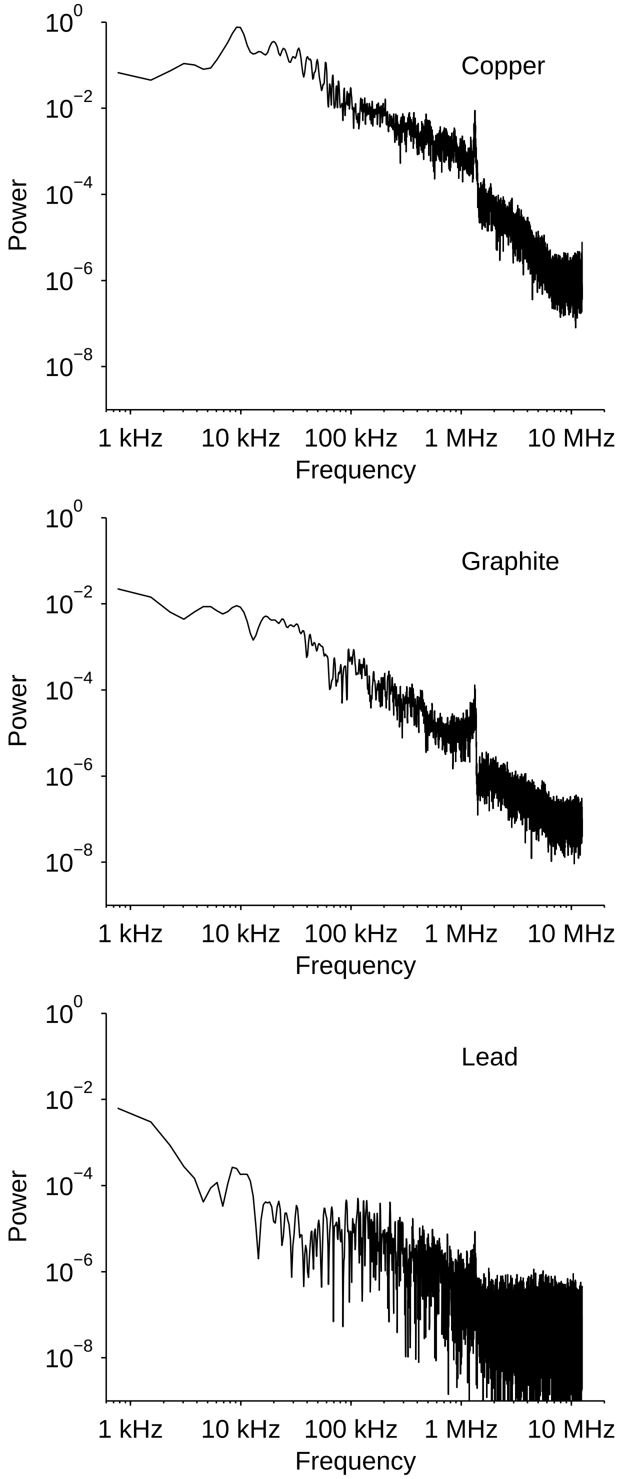

**Figure 5. Example power spectra for all three anode materials, for operation at $J^2/\dot{m} = 66$ kA$^2$-s/g.**

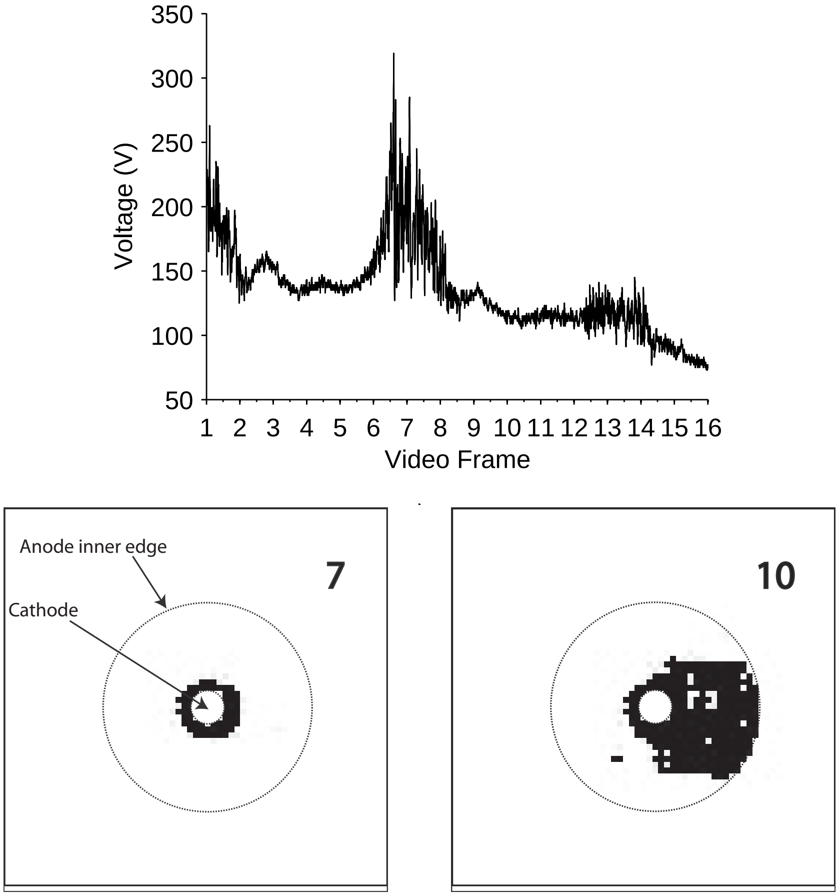

**Figure 6. 0.3 ms portion of a voltage trace at $J^2/\dot{m} = 60$ kA$^2$-s/g and two frames of a high-speed video capturing copper light emission.**

Anode damage manifests itself as individual spots or streaks distributed all over the anode surface, but preferentially at anode-insulator boundaries. The severity of the damage is a strong function of the anode material: the more refractory the material, the less damage appears. Graphite, for example, shows no evidence of significant damage for all operating conditions, even the highest $J^2/\dot{m}$; lead, on the other hand, shows significant melting for even the lowest $J^2/\dot{m}$. Aluminum and copper fall in between.

Though the damage that appears on the several anode materials we have used is of the same quality—spots and streaks, melting and discoloration—aluminum is the best material on which to make careful observations of the damage, for it visibly melts more than copper, but not as easily as lead. The damage appears in a variety of sizes on the anode surface, and we are interested in the distribution of the sizes—how many marks of each size appear. To measure the distribution of damage point sizes, we used a polished aluminum anode subjected to two above-onset firings, and surveyed the damage under optical magnification. We define the size of a mark to be the diameter for roughly circular marks, or the long dimension for oblong marks. After cataloging the damage points by size, these are placed in bins according to the sizes of the points, one bin per order of magnitude. The result is the four-point histogram shown in Fig. 7.

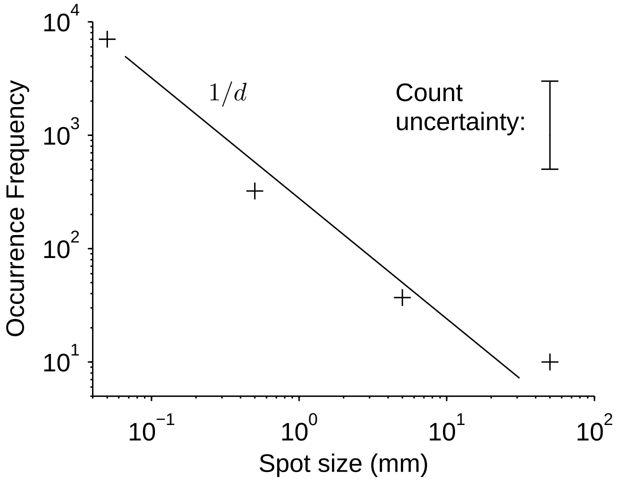

**Figure 7. Distribution of damage mark sizes from two above-onset firings on an aluminum anode.**

Within the range of damage sizes that we have catalogued, there is no preferred size. Instead, there is a distribution of sizes following the power law $1/d$, where $d$ is the spot size. The line $1/d$ is plotted along with the data points in the log-log plot of Fig. 7.

The range of spot sizes examined is limited on the large end by the largest mark size observed, which is likely related to the length of the current pulse. The limit on the low end is due to the difficulty in positively identifying damage left behind by the arc: at sizes smaller than those in the figure, marks left behind by the polishing process and, more significantly, those caused by oxidation and corrosion of the surface can appear very similar to arc damage. Investigation to smaller sizes requires a higher degree of anode surface polish than was possible in these experiments, on an anode made of a suitable metal (one that is non-corrosive in air and has appropriate thermal properties).

The negative slope of the points in Fig. 7 is artificially low, because large marks inevitably obliterate small marks. For this reason, the count in the range 1–10 mm is lower than it would be had not some of those marks been obliterated by those in the 10–100 mm range; the count is, for the same reason, too low in the 0.1–1 mm range, and so forth. While the distribution therefore has a slope close to $-1$ on the logarithmic scale shown in the figure, it is likely that the distribution of mark sizes goes like $1/d^\delta$, where $\delta \geq 1$.

That there is no characteristic damage size indicates that there is no characteristic energy delivered to the anode by a single spot—that is, no characteristic power in the spot and no characteristic spot lifetime. This is consistent with our observation that there is no characteristic timescale in the voltage hash, but rather a power-law drop in hash signal power with increasing frequency (decreasing timescale). As we might expect that a characteristic hash timescale would lead to a characteristic damage mark size, the lack of characteristic values in both these observations is satisfying.

# V. The Origin of Voltage Hash

## A. Model of Random Superposition of Events

Because the voltage hash, as we have presented it in the preceding sections, is an essentially random process, modeling based on deterministic mathematics is inappropriate. Instead, in this section we will offer an explanation for what is occurring during voltage hash based upon a random superposition of events that are likely to occur in a current-starved MPDT that is susceptible to anode spotting.

Anode spots differ from the better-understood cathode spots in that they are not the sole conductors of current to the anode, whereas arc current from a cold cathode is usually collected solely in a number of spots. Current to the anode may also be diffusely collected. In this sense, the definition of an anode spot is a local region on the anode surface of higher current density (higher conductivity) than the surrounding, diffuse regions.

Because an anode can only collect an amount of current up to the electron saturation current diffusely, any further current driven through the anode will either be conducted by spots, or will appear as a voltage rise across the anode sheath. The sheath/spot interaction can be thought of as the parallel combination of the sheath capacitance and the spot resistance. Figure 8 shows this parallel combination, and a switch whose opening and closing represent the extinction and ignition of a spot, respectively. The indicated current $I_e$ is the difference between the electron saturation current of the MPDT anode/plasma combination (diffuse current collection) and the current being driven through the thruster by the PFN ($I_e = I_{PFN} - I_{sat}$).

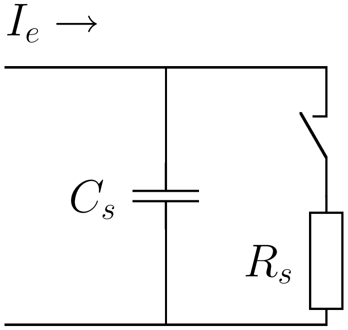

**Figure 8. Parallel $RC$ model of anode sheath and single spot.**

If the power supply is sufficiently inductive (stiff), as in our experiment, then $I_e$ is unaffected by the change in voltage across this combination as the capacitor charges and discharges. The voltage across the capacitor when the switch is opened, therefore, rises linearly with time and is

$$V_c = \frac{I_e}{C}t. \tag{2}$$

When the switch closes, the voltage across the combination falls as the capacitor discharges through the resistor, according to

$$V = V_c e^{-t/RC}. \tag{3}$$

The ratio of the rise and fall times of the voltage is $V_c/RI_e$. Experimental measures of $I_e$ are impossible to obtain, but since we do not, in general, observe voltage rise rates of greater than 100 V/$\mu$ s, and the value of the sheath capacitance is on the order of 100 nF,[25] $I_e$ is likely no larger than $\sim 10 - 100$ A. Since the resistance of the thruster plasma as a whole is on the order of 10 m$\Omega$, and $R$ should be smaller than this value, we expect that in a typical spotting cycle the rise time will be longer than the fall time. Experimentally, as well, we observe voltage fall times to be much shorter than the rise times, when we are able to distinguish individual rises and falls.

A single spotting event will therefore look something like the ramp in Fig. 9(a), where the specific values of time and voltage are dependent upon the parameters and the fall time here is shown much smaller than the rise time.

If we take this process to be indicative of what occurs across the anode sheath in the MPDT, then a full picture will include many anode spots (and the number may change with time), each having their own resistance $R$, but all sharing the same sheath capacitance $C$. The voltage that we measure across $C$

will therefore be the superposition of the activity of all the spots—or, in the parlance of our model, the superposition of many switching events.

The picture we propose is consistent with our experiment under the following conditions:

- *The current carried by each spot is sheath-limited in its steady-state.* This ensures that when a spot extinguishes, the current it had carried goes to charging the sheath capacitance, rather than being redistributed to other spots. Were this not the case, we would expect no more than one high-conductivity spot to form, inconsistent with our observations.
- *The spotting events are uncorrelated.* It is not immediately apparent, in a physical context, in what way distant spots would communicate strongly with one another—so we expect this assumption to be reasonable.

If we now take a random superposition of many events such as that in Fig. 9(a) the signal that we generate appears as that in Fig. 9(b). This particular instance is a segment of a signal generated by 4096 events randomly distributed into a time span of 16384 (in which the units are arbitrary). The rise times were randomly distributed between 2 and 512, and the amplitudes chosen randomly on the interval [0,1]. The vertical axis remains unlabeled as the amplitude of the signal can be arbitrarily scaled with the maximum event amplitude chosen. As a comparison, a segment of a real voltage trace exhibiting hash, for $J^2/\dot{m} = 72$ kA$^2$-s/g, is shown in Fig. 10.

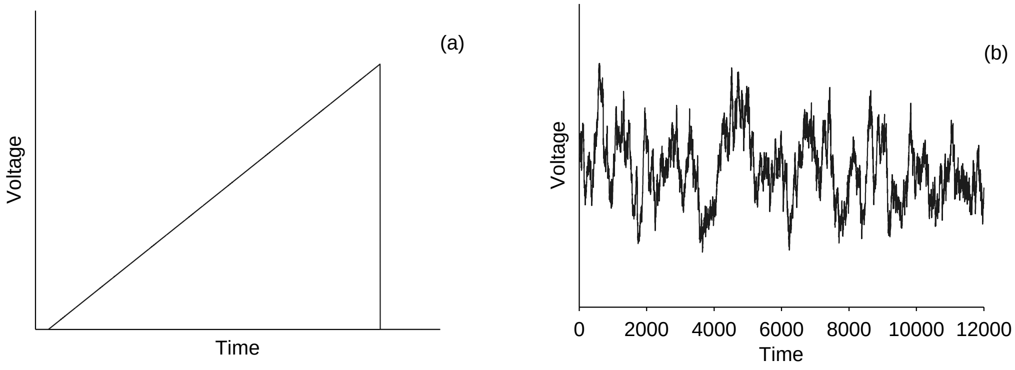

**Figure 9. The voltage change caused by (a) a single starvation/spotting event; (b) random superposition of many such events.**

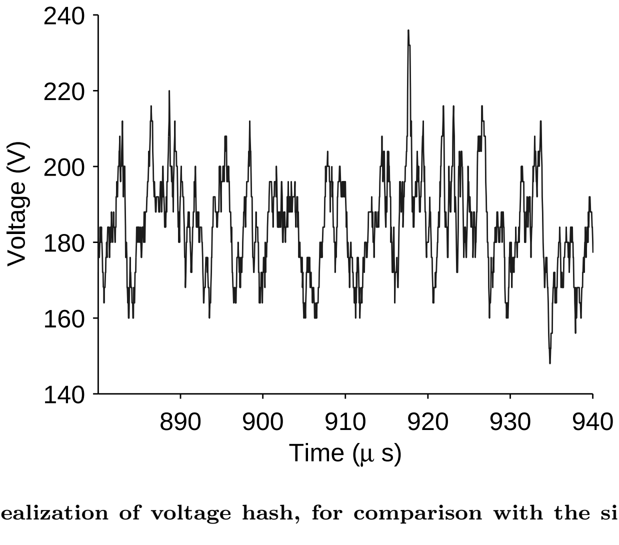

**Figure 10. Experimental realization of voltage hash, for comparison with the simulated signal in Fig. 9(b).** $J^2/\dot{m} = 72$ **kA$^2$-s/g.**

The generated signal in Fig. 9(b) is an accurate reproduction of the voltage hash. This signal is, like the voltage hash, a random walk, generated by the random superposition of deterministic functions. The power spectrum of this signal is shown in Fig. 11. The superimposed fit line on this graph corresponds to the function $1/f^{1.75}$: the power spectrum of the generated noise has a power-law drop whose exponent is, like those of the experimental voltage hash, between 1 and 2. It is clear why: by randomly superimposing many linear rises in this way, we have created a signal whose slope between any two points is a random variable. As we discussed in Sec. III.A, this gives rise to a Brownian motion such as the one we have just calculated.

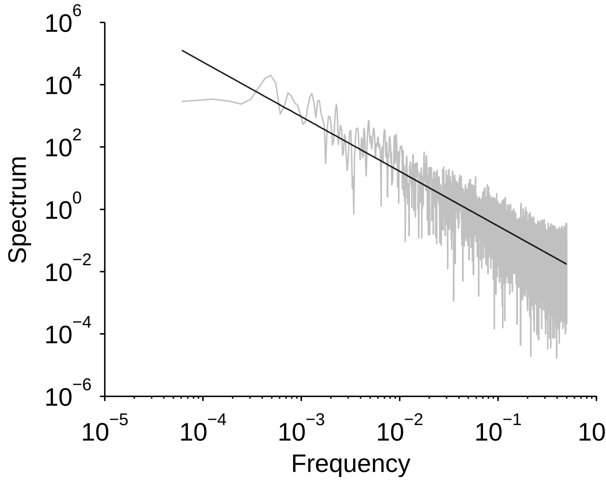

**Figure 11. The power spectrum of the signal in Fig. 9(b).**

The statistics of our generated signal depend upon the average occurrence rate $n$ of the events, and upon the rise time $\tau$ of the events. The mean and standard deviation, as we have already pointed out, can be arbitrarily scaled according to the characteristic amplitude of the events; the skewness and kurtosis of the signals, on the other hand, show a tendency toward zero as the product $n\tau$ rises—that is, the signal becomes more Gaussian as the events increasingly overlap. These trends are shown in Fig. 12. The range spanned by these statistics cover the entire range of statistics calculated for the experimental voltage hash in Sec. III.A.

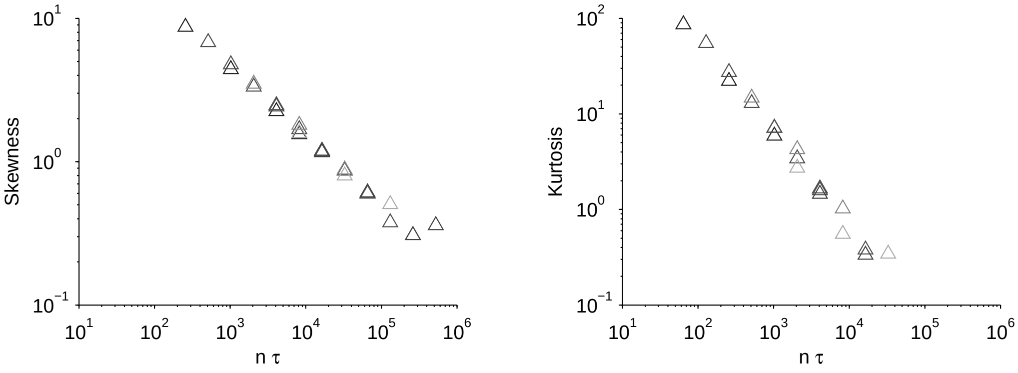

**Figure 12. Skewness (left) and kurtosis (right) of generated signals with various $n\tau$ values.**

## B. Discussion

We can use our understanding of this simple model to gain some physical insight from the experimental hash statistics. Both the skewness and kurtosis of the experimentally measured hash evolve with $J^2/\dot{m}$, both reaching a peak at the same $J^2/\dot{m}$ value. We can picture the evolution of the statistics from Gaussian, to a distribution with significant skewness and kurtosis, then back toward a Gaussian, as a movement first to the left, then back to the right, along the curves of Fig. 12. This indicates that the product $n\tau$ first decreases as $J^2/\dot{m}$ increases, and then reverses. Physically, we may understand this in the following way. The occurrence frequency $n$ is related to the average number of spots that are carrying current during a thruster firing: it is the ignition, and extinction, of these spots that create a single event. $\tau$ is representative of the current carried by a spot: the higher the current, the greater the slope of the voltage rise when the spot extinguishes, and the shorter the rise time if the voltage is not to increase to arbitrarily large values. Our data suggest that at first $\tau$ falls faster with current than $n$ rises, so that $n\tau$ falls, and the hash statistics become significantly skewed from Gaussian. At sufficiently high current, however, increasing $n$ catches up with the falling $\tau$, so that $n\tau$ falls again and the statistics return toward Gaussian.

The implication of this for our understanding of anode spotting and voltage hash can be simply stated. In order to carry increasing thruster current in the anode-starved onset condition, anode spots must either carry more current each or a greater number of anode spots must form. Our evidence suggests that in the intermediate $J^2/\dot{m}$ range, the current carried (or spot size) first grows in response to increasing thruster current; past a particular $J^2/\dot{m}$ ( $\sim 110$ kA$^2$-s/g, in our data), the number of spots then increases.

This interpretation of Fig. 4, in which we associate the return of the statistics toward Gaussian with an increasing number of anode spots, implies that an effective mechanism for anode spot creation becomes dominant at $J^2/\dot{m} \sim 110$ kA$^2$-s/g. Because this $J^2/\dot{m}$ value is independent of the anode material, we expect that this mechanism is a plasma phenomenon. It is possible that anode spots, as described here and in the references, are a manifestation of current filamentation, which is observed in coaxial plasma accelerators such as the plasma focus.[10,19] Such current filaments exhibit some of the properties that we have inferred about anode spots, such as a limit on the filament current.[10] A number of mechanisms, such as thermal instabilities and ion-acoustic instabilities, can give rise to filamentation in plasmas.[16,24] As $J^2/\dot{m}$ rises—and with it the electron drift velocity and the specific power input to the plasma—one of these instabilities may be excited, and the current will filament, increasing the number of unsteady anode spots. Should a detailed study of the relevant threshold parameters for these instabilities uncover one such threshold that is equivalent to $J^2/\dot{m} \sim 110$ kA$^2$-s/g in the FSBT, this will support our hypothesis that the mechanism behind the increasing number of anode spots after this $J^2/\dot{m}$ value is current filamentation.

## VI. Conclusion

It was our intention, in this paper, to describe the nature of voltage hash in a quantitative manner, to describe the relationship of the voltage hash to the anode damage further than we have earlier,[27] and to describe a model of the voltage hash that can explain our observations and provide some insight into the MPDT onset physics that we cannot easily observe.

We began by classifying the voltage hash as a random signal, based upon its power spectrum that lacks evidence of periodicity and the evidently non-transient nature of the signal. We calculated the statistics and power spectra of the hash for three anode materials with widely different material properties, and showed that these measures were not significantly affected by the choice of anode material. The statistics we found to evolve with $J^2/\dot{m}$, moving first away from Gaussian until $J^2/\dot{m} \sim 110$ kA$^2$-s/g, and falling back toward Gaussian afterward. The power spectra we observed to have a characteristic $1/f^{\beta}$ fall in power, as is representative of a Brownian-type motion. Non-stationarity of the statistics is due in large part to the influence of jets of anode material bridging the gap between anode and cathode.

We have also presented a model, based upon the superposition of random anode spot events, that shows the same Brownian-type motion as the experimental data, and which, depending upon the model parameters chosen, can exhibit the same statistics as the data. By associating our model parameters with the properties of anode spots—that is, the total number of spots and the current carried by the spots in a single firing—we infer from the evolution of our hash statistics that as the current through the thruster increases, the current carried by each of a small number of spots first increases, and only after that has taken place does the number of spots increase.

These insights may be applicable to attempts to control voltage hash. If, as our findings suggest, the statistics of the hash are governed by the number of anode spots and the current carried by each spot, then deliberately influencing either of these quantities should allow for control of the hash statistics. For example, an anode insulated everywhere but in a discrete number of points (so forcing the anode attachment to be in a certain number of spots) may exhibit a constrained range of hash statistics. If such a constraining of the anode attachment were to stabilize the spots, so that they did not extinguish often, this could also have the effect of suppressing the voltage hash.

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