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Edgar Y. Choueiri

Electric Propulsion and Plasma Dynamics Laboratory (EPPDyL)

Princeton University, Princeton, NJ 08544
USA

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A System Optimization Model for the Application of Ablative Pulsed Plasma Thrusters to Stationkeeping Missions*

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Abstract

We present a realistic and easy-to-apply model for the optimization of the design of an ablative pulsed plasma thruster (APPT) propulsion system for a given stationkeeping mission. By "optimization" we mean finding the design characteristics of the APPT module(s) that lead to minimum propulsion system mass for given satellite mass and mission specifications. We show that using an empirical relation for the mass production rate, an expression relating the optimal (minimum) propulsion system mass, for a given mission, to the APPT's most important scaling parameter E/I (where E is the discharge energy and I the impulse bit) can be found and, remarkably, is independent of the state of capacitor and power conditioning technologies (i.e. specific masses, conversion efficiency, etc.). This independence allows unfolding the relations between the propulsion mass, E/I, and the mission requirements in a single plot that is applicable for a wide range of Delta-v's and payload masses. The use of the model to characterize the optimal design of an APPT system is illustrated with the example of a 10-year north-south stationkeeping of a medium size commercial satellite at GEO. We show that even with off-the-shelf capacitor technology the use of an APPT system can result in propulsion mass fractions as low as 6.4%.

1 Introduction

With the prospect of the near-term deployment of small satellite constellations, the use of ablative pulsed plasma thruster (APPT) systems for stationkeeping may hold the promise of great mass savings. These mass savings, through the use of smaller launchers or the possibility of mounting multiple satellites on a single launcher, translate directly into cost savings.

We start in Section 2 with a general but realistic model for the optimal APPT system mass that includes four general types of masses: energy-dependent mass, power-dependent-mass, fixed mass and size-dependent mass. The optimized model is cast in terms of the most important scaling parameter E/I (where E is the discharge energy and I the impulse bit). In Section 3 we specialize the model with an empirical relation for the mass production rate and find that the mass of an optimized system as a function of E/I is independent of the state of capacitor and power conditioning technologies. This allows for a straightforward application of the model to a spectrum of mission requirements of current interest. In the following sections we illustrate the use of the model for the design of an APPT system optimized for specific mission requirements.

2 General Model

We can express the total mass, \( M_{\text{tot}} \), of the satellite as

\[
M_{\text{tot}} = N_m M_p + M_{\text{satp}}
\]

(1)

where \( M_{\text{satp}} \) is the mass of the satellite payload, \( M_p \) is the mass of one APPT propulsion module and \( N_m \) is the number of modules. Furthermore, \( M_p \) can be
broken down into the following components:

\[ M_p = M_{cb} + M_{pc} + M_{sp} + M_{fix} + M_{pack} \] (2)

Where \( M_{cb} \) is the mass of the capacitor bank, \( M_{pc} \) is the mass of the power conditioner, \( M_{sp} \) is the mass of the solid propellant, \( M_{fix} \) is a fixed mass (that includes the mass of the electrodes and ignitor assembly) and \( M_{pack} \) is the mass of the packaging which can be expressed as a fraction \( \epsilon \) of the total module mass

\[ M_{pack} = \epsilon M_p \] (3)

The mass of the capacitor bank mass can in turn be expressed in terms of a the discharge energy \( E \),

\[ M_{cb} = \bar{m}_{cb} E \] (4)

where \( \bar{m}_{cb} \) is the specific mass of the capacitors. The mass of the power conditioning system can be written as the sum of a fixed mass and a power-dependent mass

\[ M_{pc} = M_{pcfix} + \bar{m}_{pc} P_b \] (5)

where \( M_{pcfix} \) is the power-independent mass of that system which we include, along with other fixed masses of the system, in \( M_{fix} \). Also, \( \bar{m}_{pc} \) is the specific mass of the power conditioning system and \( P_b \) is the power required from the spacecraft bus. This power is related to the discharge energy, the pulse frequency \( f \) and the power conditioning efficiency \( \eta_{pc} \) by

\[ P_b = \frac{f E}{\eta_{pc}} \] (6)

Finally, the mass of the solid propellant can be expressed as

\[ M_{sp} = N_p F(E_i) E \] (7)

where \( N_p \) is the number of pulses and \( F(E_i) \) is the specific mass of ablated solid propellant per shot which has been shown by experiments to be a function of the ratio \( E_i \) of the discharge energy \( E \) to the impulse bit \( I \):

\[ E_i = \frac{E}{I} \] (8)

A stationkeeping mission can be characterized in terms of a Delta-v which, for a certain satellite mass, would require a total impulse, \( I_t \), from the pulsed propulsion system

\[ I_t = N_m N_p I = M_{tot} \Delta \nu \] (9)

where \( I \) is the impulse bit. This equation carries the implicit assumption that the total mass does not change appreciably over the time of the entire mission which is equivalent to saying

\[ M_{sp} \ll M_{tot} \] (10)

This assumption must therefore be checked a posteriori to validate the calculations.

Combining equations (1) to (9) we can write an expression for the mass of the APPT module

\[ M_p = \frac{\Psi \bar{m}_{bc} + M_{fix}}{1 - \epsilon - \Psi} \] (11)

where \( \Psi \) is a mass scaling parameter given by

\[ \Psi = \frac{\Delta_u}{N_p \left[ \bar{m}_{bc} + \frac{f \bar{m}_{pc}}{\eta_{pc}} + N_p F(E_i) \right] E_i} \] (12)

With the above two equations, the APPT module mass is expressed as a function of the energy to impulse bit ratio \( E_i \) which is an important parameter in scaling the APPT performance.

For a given state of the capacitor and power conditioning technology, the parameters, \( \bar{m}_{bc}, \gamma_{pc}, \eta_{pc} \) and the highest number of pulses \( N_p \) are fixed. Under such conditions there is for each value of \( E_i \) an optimal pulse frequency \( f^* \) for which \( M_p \) is minimum. We find this optimal condition by setting \( \partial M_p / \partial E_i \) to zero and solving for \( f^* \) to obtain

\[ f^* = \frac{\eta_{pc}}{\bar{m}_{pc}} \left( \bar{m}_{cb} + N_p \frac{\partial F(E_i)}{\partial E_i} \right) \] (13)

The optimal discharge energy is then given by

\[ E^* = \frac{M_p(1 - \epsilon) - M_{fix}}{\bar{m}_{bc} + \frac{f \bar{m}_{pc}}{\eta_{pc}} + N_p F(E_i)} \] (14)

Since we have expressed the mass model in terms of \( E_i \) it is also convenient and straightforward to express the following quantities in terms of \( E_i \):

the optimal specific impulse \( I_{t}^* \)

\[ I_{t}^* = [E_i F(E_i) g]^{-1} \] (15)

the total system efficiency at optimal conditions is given by

\[ \eta^* = \eta_{pc} [2 E_i F(E_i) g]^{-1} \] (16)

3 Model Specialized with a Mass Production Law

We now specialize the general model for a specific function \( F(E_i) \) obtained experimentally. It was found
in ref. [1] that the following empirical relation holds for the mass production of the propellant plasma

$$F(E_t) = E_t^{-4} / 4\kappa$$  

(17)

where \(\kappa\) is an ablation constant that depends only on the propellant. For Teflon \(\kappa = 4 \times 10^{-11}\) kg/J.

Using the above expression for \(F(E_t)\) in Eq. (13) we obtain the following expression for the optimal pulsing frequency:

$$f^* = \frac{\Delta v}{\kappa E_t^{-3}}$$  

(18)

The optimal value for the mass scaling parameter becomes

$$\Psi^* = \Delta v / \kappa E_t^{-3}$$  

(19)

It is interesting to from the above expression that \(\Psi^*\) becomes independent of all the specifications of the propulsion subsystem (specific mass, power conditioning efficiency, and number of pulses). This means that the optimal propulsion module mass, given by

$$M_p^* = \frac{\Psi^* M_{fix} + M_{fix}}{1 - \epsilon - \Psi^*}$$  

(20)

can be expressed as a function of \(E_t\) that depends only on the following four parameters: \(A_m, M_{sas}/N_m, M_{fix}\) and \(\epsilon\). The first two parameters are dependent on the mission and satellite requirements while the last two, the fixed mass and the packaging coefficient, are independent of the capacitor and power conditioning technologies.

\(M_{fix}\) can be estimated from experience at Fairchild Republic[2] as the sum of the mass of the discharge ignitor and its circuitry (23 kg), the mass of the electrodes and the associated assembly (3 kg) and the mass of the power independent part of the power conditioning system (5 kg) giving \(M_{fix} = 3.73\) kg. While the packaging ratio, \(\epsilon\), for more complex electric propulsion systems such as ion and MPD thrusters, can be as high as 0.5 (see ref. [3, 4]) APPT flight-ready prototypes have typically a packaging ratio of \(\epsilon = 0.2\) (see ref. [2]).

With fixed values of \(M_{fix}\) and \(\epsilon\), equations (19) and (20) can be used, with \(\Psi^*\) as an intermediate parameter, to build a graph that allows finding \(M_p^*\) as a function of \(E_t\) with the mission requirements \((M_{sas}/N_m\) and \(\Delta v)\) as parameters. This plot is shown in Fig. (1) for a typical range of these parameters. The use of this graph to design an optimal APPT system is illustrated below.

Figure 1: Plot relating the optimized APPT module mass \(M_p^*\) to the ratio of discharge energy to impulse bit \(E_t\) for a range of interesting mission parameters. The mass scaling parameter \(\Psi^*\) is used as an intermediate parameter to unfold the information in Eq. (19) and Eq. (20) on the same plot. The arrows across the plot show how the curves are used for the case study in Section 5. This plot is independent of capacitor and power conditioning technologies.

4 Model further Specialized with State of the Capacitor Technology

For a given state of the capacitor (and power conditioning) technology a series of curves can be drawn using the model above that give the relation between the desired average thrust and the required power from the spacecraft bus for a range of mission requirements. For our model calculations here we assume the state of the capacitor (and power conditioning) technology to be as follows:

4.1 Capacitor Specific Mass, \(\bar{m}_{c,1} = 50\) J/kg

Capacitor technology has made great advances since the early development time of the APPT's in the seventies. The capacitors with the highest energy density, \(\bar{m}_{c,1}\), that are presently available are aluminum electrolytic capacitors[5] with \(\bar{m}_{c,1}\) as high as 99 J/kg but are currently deemed unreliable for long-term space applications[3]. The double-layer capacitor technology promises a two-order of magnitude improvement in the energy density but these capacitors are currently still in the research stage[6, 7]. Barium-
strontium titanate and lead-zirconate titanate capacitors are being developed\[8\] with energy densities of $10^8$ J/kg but have not yet been shown to be suited for long-term APPT applications. Ceramic capacitors are an off-the-shelf technology and offer a factor of 2 improvement on the energy densities of the capacitors used on early APPT systems. They have been recently chosen in a pulsed MPD propulsion system study\[3\] and are benchmarked at 50.3 J/kg for the ceramic Z5U capacitor\[9\]. For our current study we set $m_{\text{start}} = 50$ J/Kg.

4.2 Power Conditioning (PC) Specific Mass and Efficiency, $m_{\text{pc}} = 8 \times 10^{-3}$ kg/W and $\eta_{\text{pc}} = 0.8$

The power conditioner (PC) for APPT systems is much simpler than that required for other EP systems and is not as critical as the capacitor bank. The power conditioner technology used at Fairchild Republic in the mid-seventies\[2\] is still well suited for today's APPT applications and is characterized by a fixed mass of 5 kg (added to $M_{\text{sys}}$ in the model above), a specific mass $m_{\text{pc}} = 8 \times 10^{-3}$ kg/W and an efficiency of $\eta_{\text{pc}} = 0.8$.

4.3 Number of pulses, $N_p = 3 \times 10^7$

The largest number of pulses demonstrated for an APPT system is in the $10^7$ range and is limited mostly by capacitor failure. High energy density capacitors have not yet been demonstrated to exceed that range so we take $N_p = 3 \times 10^7$ for our present study.

The chosen technology parameters are summarized in the middle part of Table 1.

The plot in Fig. (2) gives the required bus power (which must not exceed the available bus power) as calculated by the model above for the optimal characterization of one APPT module at a desired average thrust level for the case of $M_{\text{start}}/N_m = 250$ kg and a range of $\Delta v$ of interest. For a chosen average thrust level per module and number of modules the plot gives the required bus power for a given $\Delta v$. This will also specify the optimal value of $E_i$ ($E_i = \eta_{\text{pc}} P_i/T$) which can be used in the plot of Fig. (1) to obtain the mass of the module. Finally, the optimal pulse frequency is given by the curves in Fig. (3) obtained from Eq. (18). We note that this plot is independent of the mission requirements (i.e. $\Delta v$, $M_{\text{start}}$ and $N_m$).

We illustrate the use of these plots with the sample case study below.

5 Sample Case for the NSSK of a 500 kg Payload in GEO

A typical example from the spacecraft engineer's point of view is the case of the north-south station-keeping of a 500 kg payload in GEO for 10 years. We assume that the maximum available bus power is 300 W. The particulars of the mission (stabilization scheme, pointing requirements, stationkeeping allotments, power budgeting, etc.) dictate the choice of the number of modules and the average thrust. We assume that 2 modules are needed and each of which must deliver an average thrust of 1.5 mN. These requirements are summarized in the upper portion of Table 1.

The worst-case changes in velocity required for station-keeping at GEO are $\Delta v_{\text{MOON}} = 36.93$ m/s per year and $\Delta v_{\text{SUN}} = 14.45$ m/s per year which for 10 years yield a total of $\Delta v = 513.8$ m/s. From Fig. (2) we find, for an average thrust of 1.5 mN, a required bus power of 130 W which is acceptable since the total power for the two modules (260 W) does not exceed the available power of 300 W. This yields an energy to impulse bit ratio of $E_i = 70 \times 10^3$ J/N-s. At this value of $E_i$ (and for $N_p = 3 \times 10^7$) we find from Fig. (3) that, in order to insure the optimal conditions, the pulse frequency for each module must be .3 Hz. Furthermore, for that value of $E_i$ and the required $\Delta v$ we find from the left side of the plot in Fig. (1) a mass scaling parameter $\Psi^* = 0.4$ which, from the right side of the same plot (with $M_{\text{start}}/N_m = 250$ kg) yields an
Concluding Remarks

The APPT is the only plasma propulsion option that is currently in use on actual US spacecraft and many APPT systems have reached flight-readiness in the late-sixties and mid-seventies[2, 10]. Most of these systems, however, were limited to operation at a few tens of joules of discharge energy. For many realistic stationkeeping requirements, we have shown above that optimal APPT operating conditions (which can lead to total mass fractions of a few percent for even the more requiring stationkeeping missions) are reached for discharge energies of a few hundred joules. The scaling towards higher energy systems, the integration of modern capacitor technology, along with other technical issues[11] should be the subjects of research aiming at the development of advanced APPT systems that could meet the stationkeeping requirements of the upcoming constellations of commercial satellites.
### REQUIREMENTS

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<th>Satellite Payload Mass</th>
<th>500 kg</th>
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<tr>
<td>Mission Type</td>
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<tr>
<td>Number of Propulsion Modules</td>
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<tr>
<td>Available Bus Power</td>
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<tr>
<td>Average Thrust per module</td>
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### ASSUMED TECHNOLOGY PARAMETERS

<table>
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<th>Value</th>
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<tr>
<td>Capacitor Energy Density</td>
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<td>Power Cond. Specific Mass</td>
<td>0.5 kg + 0.8 kg/W</td>
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<tr>
<td>Power Cond. Efficiency</td>
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<td>Maximum Number of pulses</td>
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<td>Fixed Mass</td>
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<td>Packaging Mass Ratio</td>
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### OPTIMIZED APPT MODULE CHARACTERISTICS

<table>
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<tr>
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<th>Value</th>
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<td>Required Bus Power</td>
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<td>Discharge Energy</td>
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<td>Specific Impulse</td>
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<td>Mass of Capacitors</td>
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<tr>
<td>Mass of Power Cond.</td>
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<tr>
<td>Mass of Teflon</td>
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<tr>
<td>Total Mass of APPT Module</td>
<td>17.1 kg</td>
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Table 1: Sample case study: requirements and optimization results.
References


