Experiment for Plasma Energization with Beating Electrostatic Waves

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An experimental study of plasma heating by means of electrostatic waves in a magnetized, rf-sustained plasma is presented. New heating measurements are reported that unambiguously demonstrate that ion heating with beating electrostatic waves (BEW), an efficient non-resonant mechanism whose fundamental aspects and associated criteria have been analyzed theoretically [*Physical Review E*, 69(4):046402, 2004], is superior to standard single electrostatic wave heating when comparable powers are coupled into each process. Such superiority is promising for applications where plasma heating efficiency is particularly important, as in plasma propulsion. The findings were obtained from a new dedicated experiment that leverages the knowledge from previous attempts to demonstrate BEW heating. The new experimental apparatus, as well as measurements of plasma parameters, wave dispersion relations and heating levels, are described. Laser-induced fluorescence measurements show that the perpendicular ion temperature was raised by 90% over the background levels with two beating electrostatic ion cyclotron waves, compared to only 50% when using a single wave with the same energy.

Nomenclature

$v_\perp$	perpendicular velocity of magnetized ion
ω	wave frequency
k	wave number
λ	wavelength
E	wave amplitude
ω_{ci}	ion cyclotron frequency
T_e	electron temperature
T_i	ion temperature
n_e	electron density
m	mass of Ar ion
∇_r	gradient in the radial direction
∇_z	gradient in the axial direction
ν_i	Ion collision frequency

I. Introduction

The Radio Frequency (RF) heating of plasmas is a widely employed process in several industrial and scientific applications, and in recent years has seen an increasing use in electrothermal plasma propulsion.

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RF heating for this end is particularly attractive as it is electrodeless (and thus extends thruster lifetime) and potentially very efficient.[1] There are several methods by which RF heating of plasma can be achieved, but in the case of the uniformly magnetized plasma found in many electrothermal plasma propulsion concepts, heating by means of non-linear wave-particle interactions with perpendicularly-propagating electrostatic waves shows particular promise. Indeed, Karney[2] theoretically demonstrated in 1977 that in the presence of perpendicularly-propagating electrostatic waves, magnetized ions with perpendicular velocity in the so-called resonance zone ($v_\perp \sim \omega/k$) are subject to significant non-linear acceleration. Therefore, single electrostatic wave (SEW) heating can be achieved in an actual magnetized plasma by means of the non-linear energization of a small population of ions resonant with the perturbing electrostatic wave and the subsequent distribution of the gained energy to the rest of the ion population through collisions. The efficiency of this process is bounded, however, by the fact that the energy exchange of the wave is limited to a small fraction of the ion population in the plasma.

Benisti *et. al*[3] found in 1998 that this limitation can be overcome by using two electrostatic waves. In this case, ions outside of the resonant zone are accelerated, and indeed, ions with arbitrarily low initial energy can be accelerated by two electrostatic waves that satisfy the so-called beat criterion, $\omega_1 - \omega_2 = n\omega_{ci}$, where ω_1 and ω_2 are the frequencies of the exciting waves, ω_{ci} is the cyclotron frequency of the ions, and n is a positive integer.[3] Subsequently, Choueiri and Spektor[4] showed theoretically that not all ions are subject to acceleration in the beating electrostatic wave (BEW) process and then derived[5] an additional criterion, cast in terms of an inequality that the ion's Hamiltonian must satisfy, for BEW acceleration to occur. Spektor and Choueiri,[11] and more recently Jorns and Choueiri[6] extended the analysis of single ion acceleration to that of a plasma and demonstrated, also theoretically, that generally for an initially Maxwellian plasma, the existence of a population of ions that satisfy this additional criterion will lead to plasma heating levels that are significantly higher than those attainable with SEW heating.

In light of these theoretical findings, Spektor and Choueiri[7] in 2005 constructed the Beating Waves Experiment (BWX) to explore the degree of heating that occurs for SEW and BEW excitation of a uniformly magnetized, RF-sustained, axially symmetric plasma. They reported for the case of $n = 1$, a $15 \pm 12\%$ increase in ion temperature over background for SEW heating and a $35 \pm 13\%$ increase for BEW heating. While these results showed statistically significant heating, within the error bars there was uncertainty as to whether or not the BEW scenario was superior. Thus, while BWX did demonstrate that BEW heating does occur, it did not conclusively prove its improvement over single wave heating. Moreover, these results could not be improved upon in that earlier setup as high wave damping in the plasma, ambient noise from the RF plasma source, and poor heating antenna impedance matching significantly limited the heating efficiency.[8]

In order to conclusively demonstrate the superiority of BEW heating then, the need is apparent for a next generation experiment. We outlined the design of such an experiment[9] and have recently completed its construction. The goal of this paper is to describe the first experimental investigation conducted with this second-generation experiment (BWX II) and to attempt to resolve the ambiguity about BEW versus SEW heating. To this end, this paper is structured in the following way. Section II outlines the experimental setup of BWX II. Section III describes the experimental parameters observed during the heating investigation including both plasma and wave propagation properties. Sec IV presents the ion heating results. And finally, in Sec. V we discuss the implications of the heating results and future modifications to be made to this experiment.

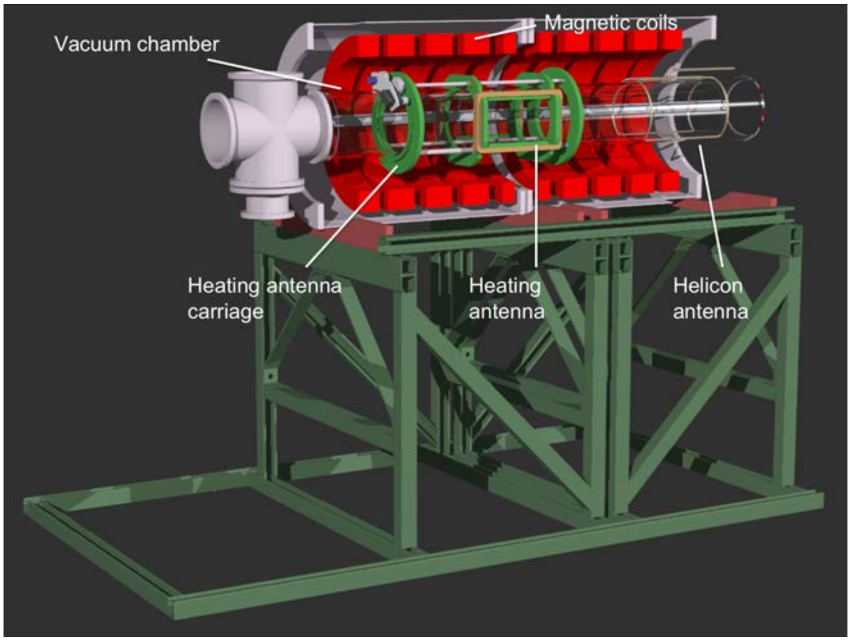
II. Experimental Setup

The full design of the BWX II is detailed in Ref 9; however, a brief summary of the components is provided here.

A. Vacuum Chamber and Solenoid

A rendering of the experimental apparatus is shown in Figure 1. A single Pyrex cylinder 52" in length with a 6.5" inner diameter is placed concentrically in a 48" long, 10 ring solenoid. A small window placed at the end of the chamber provides longitudinal optical access while argon gas flows into the chamber through a feed in the cross at the opposite end of the chamber. A constant pressure of 0.1 to 30 mTorr is maintained by a 140 l/s turbo pump with a conductance controller as well as a roughing pump.

Once the discharge of plasma is created, the plasma propagates along the magnetic field lines into the



(a) Rendering of the experimental apparatus

(b) Photograph of the experimental apparatus showing the LIF system in the foreground.

Figure 1. Rendering of the experimental apparatus.

experimental region. The solenoid consists of two Klystron Varian 1955A magnets placed end to end. While each magnet is rated to produce fields up to 1000G, we used a magnetic field of 688G for this investigation.

B. Plasma source

A Boswell type saddle antenna with a 7.25" inner diameter is placed around the vacuum chamber at one end of the solenoid as shown in the photograph in Figure 2 . The antenna is actively water cooled and produces an inductive discharge by means of an ENI 13.56 MHz 1.28K power supply. The source is impedance matched to the plasma with an L network consisting of two Jennings 1000 pF 3kV variable vacuum capacitors and was operated at 350W for this experimental investigation. The antenna is positioned 18" away from the test region in order to minimize interference from the plasma source during ion heating measurements. This is a significant improvement on the first generation BWX as the plasma source was in the immediate vicinity of the test region.

Figure 2. Photograph of one end of the experiment showing the plasma and part of the Boswell type saddle antenna wrapped around the vessel.

C. Heating Antenna

A rendering of the the heating antenna is depicted in Figure 3. The antenna is a transverse Helmholtz configuration that is modeled after the extremely successful rectangular antenna geometry employed by Kline for single electrostatic wave heating.[10] It is a 6" × 9" rectangle and consists of 20 loops on each coil mounted directly on the pyrex vacuum vessel. The coils are operated in phase from 2kHz to 20 MHz and powered with an ENI 100 W amplifier. At maximum power and a frequency of 50.6 kHz (the frequency used in this experimental investigation for SEW heating), the RMS magnetic field at the center of the antenna was measured to be 15 G.

Currently, there is no circuit element for impedance matching the antenna. Instead, we scanned the frequency range until the maximum amount of power was coupled into the plasma. It also should be noted that this antenna configuration was not possible on the first generation BWX as there was a 3" vacuum gap between the plasma and vacuum chamber walls. This prevented the inductive coupling between the plasma and antenna that occurs easily in BWX II.

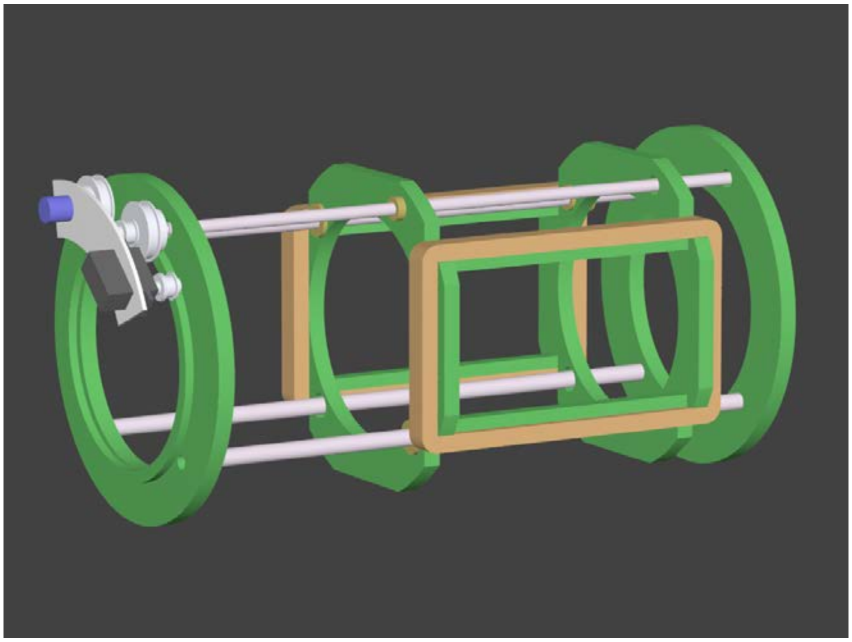


Figure 3. Computer rendering of the Helmholtz transverse antenna and its carriage.

III. Experimental Parameters

As outlined in brief in the previous section, the plasma source was operated in the inductive mode at 350 W in a 688G magnetic field. The fill pressure of argon was measured to be 2 mT. These experimental parameters were chosen for the investigation as we observed the highest levels of ion heating at these values. In the following section we discuss the diagnostics and measured results for the plasma parameters.

A. Density and Temperature Measurements

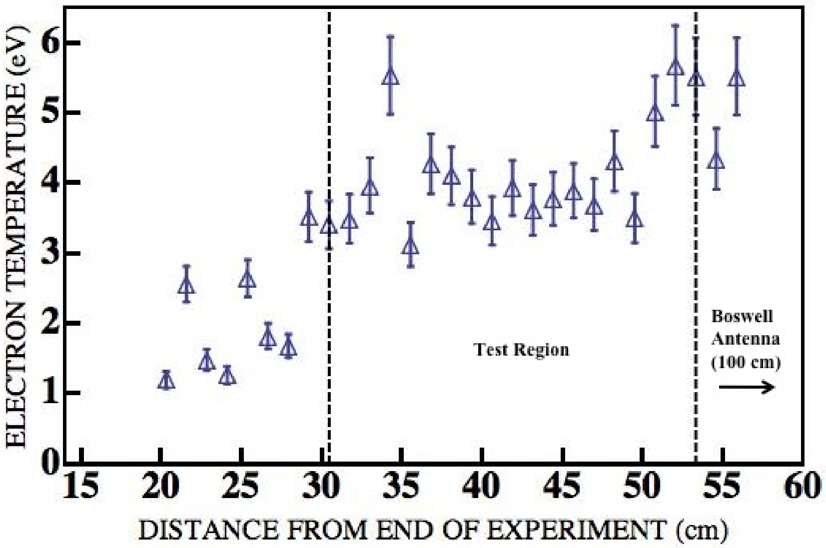
In order to take density and temperature measurements, we employed an RF-compensated voltage-swept Langmuir probe. The axial and radial profiles for electron temperature are shown in Figure 4(a) and Figure 4(b) along with a depiction of the bounds of the experimental region. In the axial graph, the distance reported is measured from the end of the experiment opposite to the RF plasma source. As can be seen, as the probe is moved closer to the RF source, the potential drop results in a gradual increase in electron temperature. However, as can be seen from Figure 4(a), in the experimental heating region the electron temperature is relatively constant at an average value of 3.5 - 4 eV. In the radial direction there is a more pronounced decrease in electron temperature that was also reported in the first BWX.[8]

Despite the gradient in these values, the plasma can still be considered uniform on the length scales of interest. Indeed, as will be discussed in Sec IV, the observed background ion temperature in this plasma is on the order of 0.5 eV. For the experimental magnetic field of 688 G, this corresponds to an ion cyclotron radius of approximately $r_i = 0.2$ cm. We thus can see that the gradient in the axial and radial directions respectively is such that everywhere in the experimental region

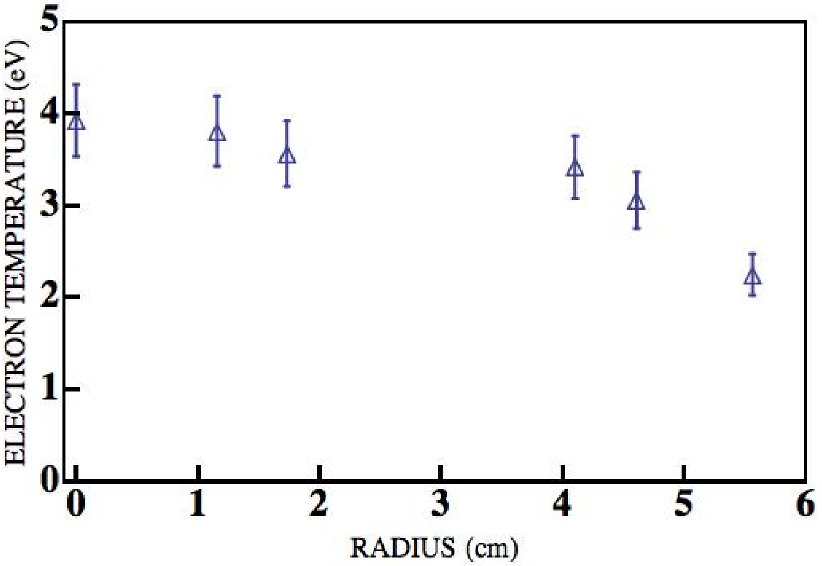
$$\frac{\nabla_r T_e}{T_e} r_i << 1, \qquad \frac{\nabla_z T_e}{T_e} r_i << 1, \tag{1}$$

where $\nabla_r T_e$ and $\nabla_z T_e$ represent the gradients in the radial and axial positions respectively. Since these inequalities hold, we thus can consider that the plasma temperature is uniform on the length scale of ion cyclotron motion. Furthermore, as we will discuss shortly, the excited waves in the plasma are measured to have wavelengths such that $\lambda \sim 1$ cm. As a consequence, we see that the relationship

$$\frac{\nabla_r T_e}{T_e} \lambda << 1, \qquad \frac{\nabla_z T_e}{T_e} \lambda << 1, \tag{2}$$

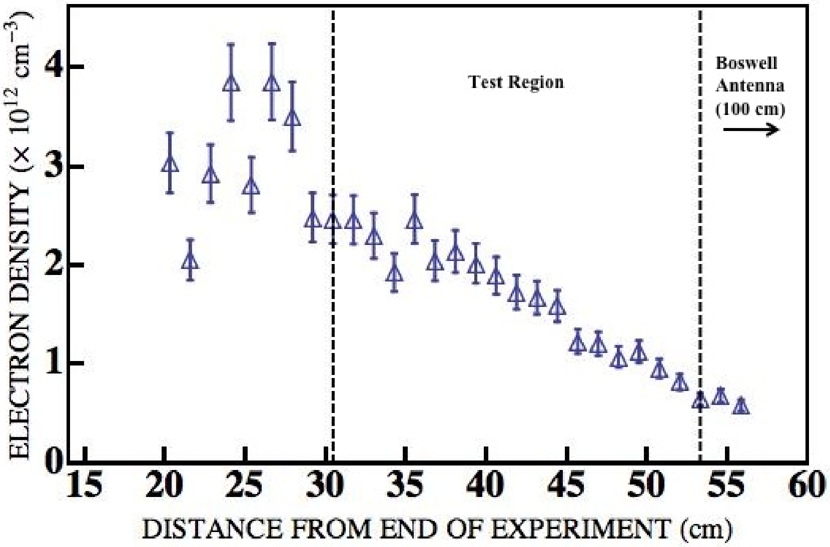


(a) Axial dependence of electron temperature in the BWX II setup. The region between the dotted lines represenst the location of the heating antenna and where all wave and ion heating measurements are performed. The distance is measured from the end of the experiment opposite the RF plasma source.

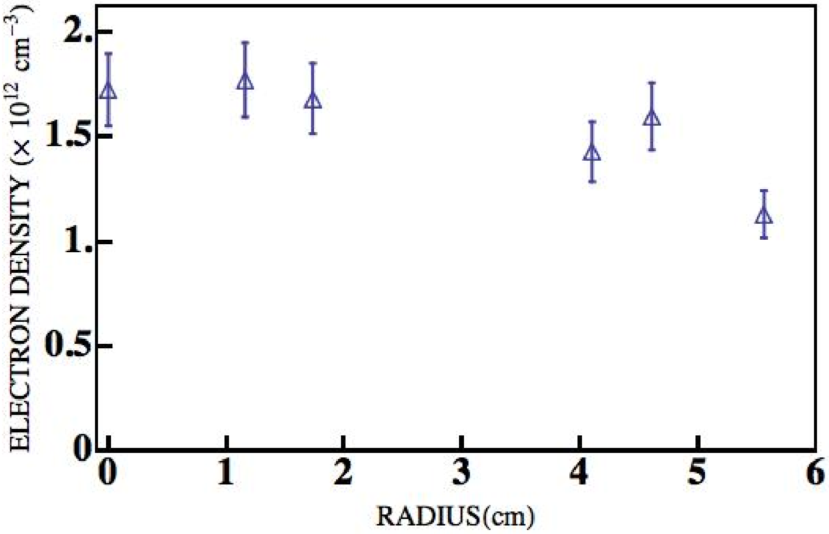


(b) Radial dependence of electron temperature in the BWX II setup measured at the center of the experimental region (~43 cm)

Figure 4.



(a) Axial dependence of electron density in the BWX II setup. The region between the dotted lines represenst the location of the heating antenna and where all wave and ion heating measurements are performed. The distance is measured from the end of the experiment opposite the RF plasma source.



(b) Radial dependence of electron density in the BWX II setup measured at the center of the experimental region (∼ 43 cm)

Figure 5.

also holds and the plasma temperature is also considered uniform on the wavelength scale in the plasma.

The axial and radial dependences of electron density are shown in Figure 5(a) and Fig 5(b). While the density is relatively uniform in the radial direction, there is a significant gradient in density observed along the axis of the experiment. Despite this non-uniformity, however, we can follow the treatment outlined above for the electron temperature and find that

$$\frac{\nabla_r T_e}{n_e} r_i << 1, \qquad \frac{\nabla_z n_e}{n_e} \lambda << 1, \tag{3}$$

$$\frac{\nabla_r n_e}{n_e} r_i << 1, \qquad \frac{\nabla_z n_e}{n_e} \lambda << 1, \tag{4}$$

where $\nabla_r n_e$ and $\nabla_z n_e$ represent the gradients in electron density in the radial and axial positions respectively. As these relationships hold, we can conclude just as in the electron temperature case that the plasma density is uniform on the ion cyclotron and wave length scales.

Therefore, we conclude that the electron temperature and density variations are sufficiently mild to permit a uniform plasma approximation on the wave and ion motion length scales of the plasma. As outlined in Refs. 3, 7, 11, this is necessary assumption to make for the existing SEW and BEW heating theories to be valid.

B. Electrostatic Wave Dispersion Relation

In order to produce SEW and BEW heating, it is necessary that the frequency of wave excitation be in the frequency range of a natural electrostatic mode of the plasma, such as the ion cyclotron frequency ω_{ci} or the lower hybrid frequency. In the ion cyclotron range, as outlined in Ref. 7, there are two electrostatic modes that can be launched perpendicularly to the magnetic field, the Neutralized Ion Bernstein Mode and the Electrostatic Ion Cyclotron Wave (EIC). However, due to the high collisionalitty of our BWX II plasma ($\omega_{ci}/\nu_i < 0.1$), the Neutralized Ion Bernstein Wave is heavily damped and does not propagate. This implies that most of the energy coupled into the BWX II plasma may be available to EIC waves. The theoretical dispersion for this mode is given by[12]

$$\omega^2 = \omega_{ci}^2 + k^2 \frac{T_e}{m}, \tag{5}$$

where k is the wavenumber and m is the mass of the ion species. In order to measure the experimental dispersion relationship in the BWX II plasma, we followed the procedure outlined in Ref.[13] In brief, we launched a single wave in the plasma at a known frequency and inserted a biased RF probe into the plasma to measure the trace of the perturbations caused by the exciting waves. We then measured the signal at a separate radial distance and determined the time delay between the two traces. With this time delay Δt, the frequency of excitation ω, and the known distance between the points of measurement d, we then were able to determine the wavenumber k of the exciting waves with the relationship

$$k = \frac{\Delta t}{d} \omega. \tag{6}$$

This procedure enabled us to measure successfully the wavenumbers in the plasma for a wide range of frequencies. This empirically observed dispersion relation along with a theoretical fit using Eq. 5 is shown in Figure 6. As can be seen, the observed values correspond quite well to the theoretical curve–thus corroborating the assertion that EIC waves are propagating in the plasma. As a separate check, the best fit curve corresponds to an electron temperature of 3.9 eV. This matches quite well the T_e values reported in the experimental region of the plasma in the previous section.

IV. Ion Heating with Electrostatic Waves

In order to measure ion temperature in the BWX II plasma, we employed a Laser Induced Fluorescence system shown schematically Figure 7. The system has five components: a tunable diode laser centered at 668.6130 nm, a wavemeter, a signal chopper, a Stanford Lock-In Amplifier, and a collection optics lens stationed orthogonal to the incident laser beam. The central wavelength of the laser is tuned to the $3d^4F_{7/2} - 4p^4D^0_{5/2}$ transition of the metastable state of ArII which decays to the $4s^4P_{3/2}$ state producing 442.72 nm

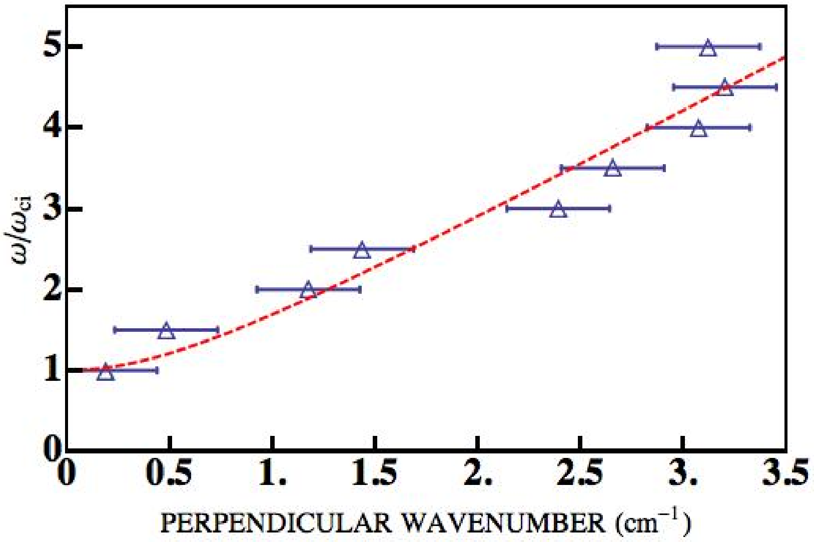


Figure 6. Empirically observed dispersion relation for electrostatic waves in the BWX II plasma. The dotted line is a theoretical best fit using the EIC wave dispersion relation. The frequencies are normalized to the cyclotron frequency ω_{ci} that corresponds to a magnetic field of 688 G.

light. The wavelength of the beam is swept by 0.2 nm in the red and blue directions around the central wavelength, and the subsequent intensity profile of fluoresced light $I(\nu)$ is recorded.

All LIF ion temperature measurements were performed in steady state with either the antenna activated or off. Before we attempted to take any heating measurements, however, we first established the background ion temperature in the experimental region. Repeated measurements revealed that the ambient temperature was $T_i = 0.54 \pm .07eV$. This value was confirmed before each heating trial.

For the actual heating experiments, since we do not yet have an impedance matching circuit for the heating antenna as mentioned in Sec. II, we were forced to scan the ion cyclotron frequency range for a suitable value that corresponded to the best impedance match with the plasma. We ultimately found the best match at approximately $\omega = 1.92\omega_{ci}$. For SEW heating experiments then, we produced a sinusoidal function with this frequency and amplitude E on the transverse heating antenna. After recording the subsequent increase in temperature, we then switched to BEW heating. For this trial, we mixed two waves with frequencies of $1.92\omega_{ci}$ and $2.92\ \omega_{ci}$ and each with amplitude $E/\sqrt{2}$ and excited them at the antenna. This latter choice of amplitude was made to insure that comparable power was being delivered to the plasma in both the SEW and BEW case.

The results of our first heating measurements as well as those from the original BEW are shown in Figure 8. As can be seen, the BEW heating is $\sim$ 90% while the SEW heating is $\sim$ 50%– nearly an improvement of a factor of 3 over the previous BWX. Moreover, it is readily apparent that the increase in heating over SEW due to the BEW process is well outside the margin of error. This demonstrates that BEW heating, even under the un-optimized conditions of these first experiments, can be unambiguously superior to SEW heating.

V. Discussion

The results from the previous section are a consequence of the improved BWX II setup. In particular, the increased heating stems in large part from the the new antenna configuration that permits improved coupling to the plasma and the reduction in noise achieved by increasing the distance between the experimental region and plasma source. We believe that if we can develop an improved impedance matching circuit for the transverse antenna, the improved coupling will lead to even more pronounced heating. This may be achieved through an L-network of capacitors similar to the setup employed for our RF plasma source.

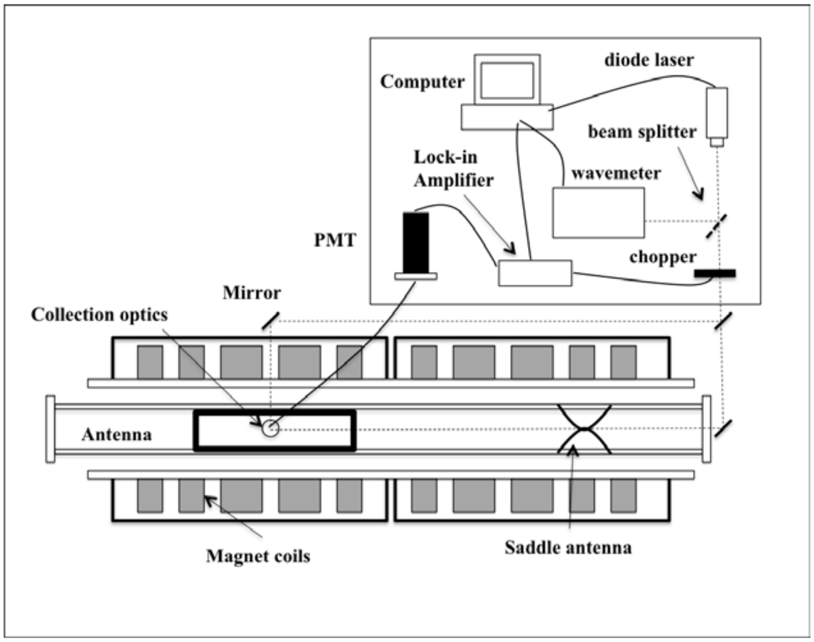


Figure 7. Schematic of the Laser Induced Fluorescence system. The dotted line indicates the laser beam path. There are two possible paths, longitudinal and perpendicular to the field lines for temperature and wave measurements perpendicular to and along the magnetic field. The system can only view one direction at a time.

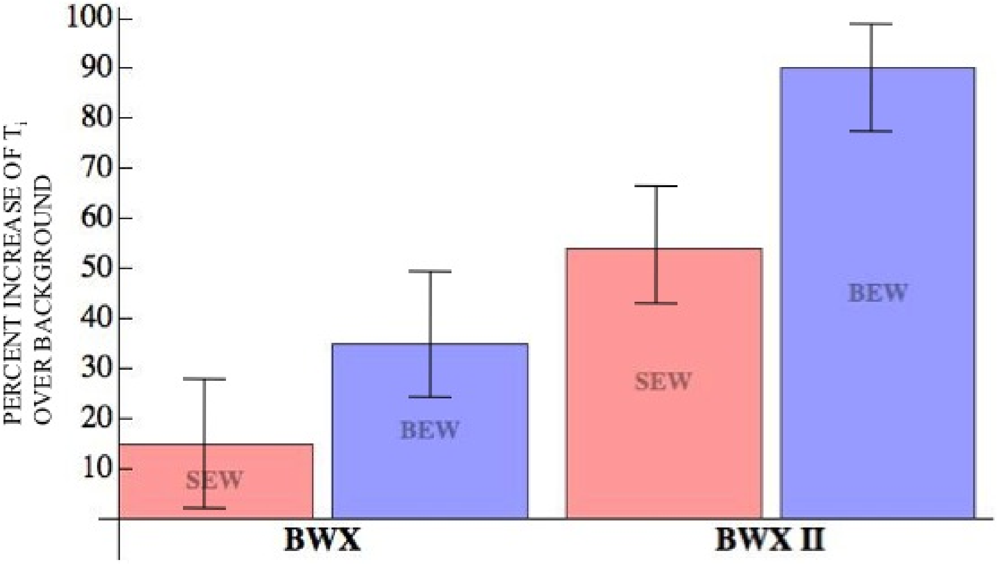


Figure 8. Representation of the maximum heating results produced by the first generation BWX and BWX II.

For the time being, however, these initial results from the newly minted BWX II serve as an encouraging validation of continued investigation into non-resonant heating with Beating Electrostatic Waves. BWX II has the diagnostic capability to further investigate into the frequency and amplitude dependence of this heating mechanism. This is a study that will be done in conjunction with efforts to further improve the level of heating.

VI. Conclusion

In this paper, we have outlined our experimental investigation into electrostatic heating by means of new cylindrical, uniformly magnetized, RF-discharge plasma. We have identified several improvements in this new setup over the first generation BWX and how they can lead to improved heating. We further have successfully characterized the plasma properties of the BWX II and identified that we do in fact excite Electrostatic Ion Cycltron Waves in the plasma.

With these waves we chose an appropriate exciting frequency to maximize coupling between the plasma and antenna and were able to show a a 50% increase in ion temperature over the background level for SEW heating and a 90% increase for BEW heating. We thus have conclusively demonstrated experimentally that BEW heating can be superior to SEW heating if comparable levels of power are introduced to the plasma through the waves. As a final observation, we also identified a possible method to improve heating for future investigation that centers upon better tuning the heating antenna. With this in mind, these reported results have successfully confirmed the potential for BEW heating and prepared the way for an extensive experimental characterization of this heating process.

VII. Acknowledgements

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