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Technical Notes

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Corruption of Pulsed Electric Thruster Voltage Fluctuation Measurements by Transmission Line Resonances

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Nomenclature

C = ladder network stage capacitance
E = difference operator
L = ladder network stage inductance
ℓ = number of ladder network stages
m = voltage measurement node
n = ladder network node identifier, $0 \leq n \leq \ell$
R_t = termination resistance
s = Laplace variable
$V(t)$ = transmission line driving signal
$\tilde{V}_n$ = Laplace transform of the voltage at any node n
x = length of transmission line
$\lambda_{\min}$ = smallest wavelength present in $V(t)$
ω = oscillation frequency

I. Introduction

THE measurement of fluctuations in the electrical characteristics (voltage and current) of electric thrusters is a valuable diagnostic technique for studying various aspects of thruster behavior and deriving insight into the physical processes behind these fluctuations. Measurements of voltage and current fluctuations are not influenced only by arc behavior, but can be significantly corrupted by the action of the external power-supply circuitry; therefore, power-supply effects can alter the physical interpretation of fluctuations if not taken into account. Unlike steady-state Hall thrusters, arcjets, and magnetoplasmadynamic thrusters (MPDTs), which are connected to power processing units controlled by nonlinear and active components, pulsed thrusters (e.g., quasi-steady magnetohydrodynamic, pulsed plasma, and pulsed inductive thrusters) are powered by only passive components, making them a particularly insightful case with which to study power-supply corruption.

In this Note, we study the corrupting effect of the power transmission line on the measurement of transient voltages across a pulsed thruster with a passive power supply. (This analysis can also be extended to the case of transient currents.) We start with the analysis for the case of an arbitrary thruster and transmission line in Sec. II. Then, in Sec. IV, we use the case of a pulsed MPDT (whose system design makes measurements of voltage transients particularly prone to transmission line corruption) as an experimental example of the extent to which the power transmission line can change the nature of voltage measurements.

II. Analysis of Cabling Resonances

In this Note, we derive the response of a transmission line to a general voltage excitation in a form useful for our particular discussion. We model a lossless transmission line, as shown in Fig. 1, a ladder network of ℓ inductors and capacitors that approximate distributed parameters as their numbers increase and their values decrease, keeping the total inductance and capacitance (ℓL and ℓC) constant. For this model to provide an accurate approximation to a real transmission line, the length of line lumped into a single L-C station must be short compared with a quarter-wavelength of the highest frequency in the general driving signal $V(t)$ [1]. The restriction this places on the number of L-C stations is $\ell \gg 4x/\lambda_{\min}$, where x and $\lambda_{\min}$ are the length of the line and the smallest wavelength in $V(t)$, respectively. Values of L and C are then chosen so that ℓL and ℓC equal the inductance and capacitance of the line being considered.

We derive the response of the transmission line using the relationship between the voltage at any two adjacent nodes and the current flowing through any two adjacent inductors:

$$V_n - V_{n+1} = L\frac{\mathrm{d}I_{n+1}}{\mathrm{d}t} \qquad I_n - I_{n+1} = C\frac{\mathrm{d}V_n}{\mathrm{d}t} \tag{1}$$

These first-order equations are purely algebraic in Laplace space; therefore, we take the Laplace transform of each and combine the two to obtain a single equation relating the voltage at node n to that at the two adjacent nodes. The resulting equation is a first-order difference equation in n,

$$\tilde{V}_{n+1} - (2 + s^2LC)\tilde{V}_n + \tilde{V}_{n-1} = 0 \tag{2}$$

whose characteristic equation in the difference operator E is

$$[E^2 - (2 + s^2LC)E + 1]V_n = 0 \tag{3}$$

The roots of the characteristic equation are

$$E_{1,2} = 1 + \frac{s^2LC}{2} \mp s\sqrt{LC}\sqrt{1 + \frac{s^2LC}{4}} \tag{4}$$

so that the solution to Eq. (2), giving the voltage at any node n, is

$$\tilde{V}_n = AE_1^n + BE_2^n \tag{5}$$

with A and B constants in n (though not in s). The values of these constants are found by application of the boundary conditions. The first of these is at node $n = 0$, at which the voltage is equal to the applied value, so that

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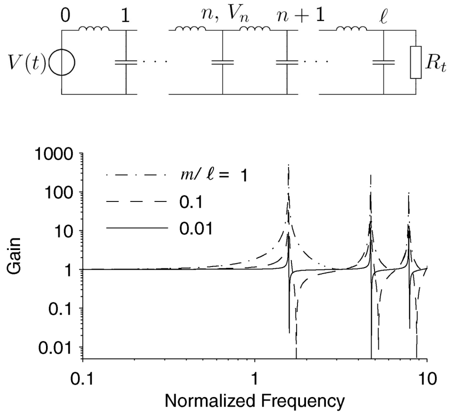

Fig. 1 (Top) The lumped-element circuit used to approximate the response of a transmission line $V(t)$. (Bottom) Transfer function gain $|\tilde{V}_n/\tilde{V}_0|$ from Eq. (10) vs the normalized excitation frequency $\omega\sqrt{\ell^2 LC}$.

$$\tilde{V}_0 = A + B \tag{6}$$

The second boundary condition states that the voltage at node $n = \ell$ must be equal to the voltage across the terminating resistance R_t:

$$\tilde{V}_\ell = R_t(\tilde{I}_\ell - sC\tilde{V}_\ell) \tag{7}$$

With the simplifying definition $\kappa \equiv 1 + sL/R_t + s^2LC$, this boundary condition gives the ratio

$$\frac{A}{B} = -\frac{\kappa E_2 - 1}{\kappa E_1 - 1}\left(\frac{E_2}{E_1}\right)^{\ell-1} \equiv \phi \tag{8}$$

so that

$$A = \frac{\phi}{1+\phi}\tilde{V}_0 \quad \text{and} \quad B = \frac{1}{1+\phi}\tilde{V}_0 \tag{9}$$

Finally, the relationship between the input voltage $\tilde{V}_0$ and the voltage at any node n $\tilde{V}_n$ is

$$\frac{\tilde{V}_n}{\tilde{V}_0} = \frac{1}{1+\phi}(\phi E_1^n + E_2^n) \tag{10}$$

The solution in Eq. (10) accurately reproduces such phenomena as cable resonances due to standing waves, termination phenomena, and phase shift. Because it allows a simple calculation of the voltage at any node n, it readily facilitates our discussion of the transmission line's effect on voltage measurements.

III. Application to Thruster Voltage Measurements

Any pulsed electric thruster is connected to its power supply through some length of transmission line that can be modeled as in the previous section. The thruster itself is the voltage source $V(t)$ to be measured, and the thruster voltage measurement must be located at some point m along the transmission line. Physical constraints often dictate that this point be some distance from the thruster. The measurement is therefore not, practically speaking, taken at node 0, where it ideally ought to be, but at some node $0 \le m \le \ell$. The terminating resistance R_t is a large value; the transmission line is essentially open-circuited when connected to the passive supply. We will now discuss the important consequences of this arrangement.

The plot in Fig. 1 shows the transfer function of Eq. (10) relating a voltage measurement taken at various values of m/ℓ to the input voltage V_0. The frequency ω ($=is$) is shown normalized to the L-C resonance frequency of the total line parameters ℓL and ℓC. This plot shows that transmission line resonances appearing in the transfer function significantly emphasize certain frequency components of the input signal, even for voltage measurements made relatively close to the source (e.g., $m/\ell = 0.01$). The bandwidth of affected frequencies and the magnitude of the resonances are both decreased as $m/\ell \to 0$, and the transfer function approaches the ideal of a flat line with unity gain. The most important insight to be gained from Eq. (10) and Fig. 1 is the extent to which a measurement of $V(t)$ can be altered by a measurement location, even at a small distance away from the source along the transmission line.

IV. Experimental Verification

The pulsed MPDT is particularly susceptible to the problem discussed in the last section. It typically is powered by a high-energy pulse-forming network that, because of practical constraints, is located some distance from the thruster; relatively long pulse times, on the order of 1 ms, do not require particularly low-inductance configurations. The cabling is therefore usually long and the voltage measurement easier to take at a point distant from the thruster. In this section, we demonstrate the principles of the last section using a quasi-steady MPDT [2,3]. The pulsed MPDT voltage contains large transients during operation at high currents, at and above the "onset" current [3]. The power transmission line for this particular thruster is ~11 m of 40 parallel RG-8 coaxial cables and other parts (e.g., gas switch, thrust stand, ballast resistor), whose combined inductance and capacitance place the fundamental resonance frequency at 1.3 MHz.

For this demonstration, we simultaneously measured the voltage at two locations with identical voltage probes, the first (the "inner" measurement) at 7.5 cm behind the upstream end of the discharge chamber (the closest feasible location to the MPDT discharge) and the second at 1 m from the thruster body, at the power feedthrough on the vacuum tank (the "outer" measurement), where such measurements are commonly taken. The m/ℓ values for these two locations are 10^{-4} and 0.05, respectively. These values were calculated by taking the ratio of the inductance between the discharge and the measurement location (which, in the first case, included only the thruster body, and in the second, the thruster body, thrust stand, and vacuum feedthrough) to the inductance of the entire power transmission line. This method of calculating m/ℓ is necessary because the distributed capacitance and inductance of the various components are not equal to one another, as assumed in the model of the last section; however, because the physical extent of these components is much smaller than wavelengths of interest in the voltage signal, their inductance and capacitance contributions can be lumped into L-C stations in the model of Sec. III in the same way as lengths of constant-parameter transmission line.

We show, in Fig. 2, a comparison between the inner and outer voltage measurements, for the case of the MPDT firing at 22 kA with 4.5 g/s argon propellant feed. This is a condition well above the onset current (14 kA), and significant voltage transients appear on both traces. However, the most prominent feature on the outer measurement is a large-amplitude sinusoidal *oscillation* around a mean of 350 V with a frequency of 1.3 MHz, whereas what appears most prominently on the inner measurement is a series of voltage *spikes* rising several hundred volts from a 350 V baseline, each lasting several μs. These spikes (which are believed to be signatures of anode spots [4,5]), and not the oscillations, represent the true nature of the fluctuating voltage.

Power spectra of the two measurements, each showing the effect of the resonance, are also shown in Fig. 2. The peak of the resonance on the inner measurement is six times smaller, and much narrower, than that of the outer measurement. Each of these empirical observations was predicted by the trends seen in Fig. 1. Because the resonance on the outer measurement is so large and broad, attempts to filter out the 1.3 MHz oscillation would affect frequencies well down into the hundreds of kilohertz, leaving the filtered waveform bearing only a loose resemblance to the true waveform.

V. Conclusions

The discussion of analytical and experimental results in this Note demonstrates the importance of locating the voltage measurement as close as possible to the thruster in pulsed thruster experiments where voltage oscillations are of interest. Some pulsed thrusters may avoid

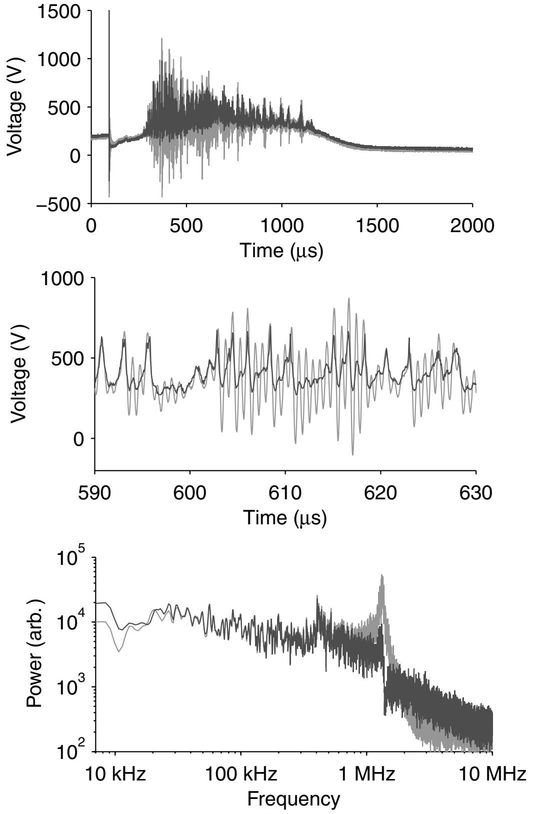

Fig. 2 Above-onset quasi-steady MPDT voltage. In the full trace (upper), expanded trace (middle), and power spectra (bottom), the gray trace is the "outside" and the black trace is the "inside" measurement.

this problem altogether by their low-inductance design: with no, or a very short, transmission line between power source and thruster, line resonances occur at such high frequencies that they lie outside the time scales of interest [6,7]. For other pulsed plasma thrusters, such as nonoptimized laboratory models, which do have transmission lines [8], the analysis of this Note may be used to determine if the resonances will significantly corrupt the measurement of voltage transients. Eliminating the resonances may then be carried out by either suitably relocating the voltage measurement, or by accurately measuring the transfer function of Eq. (10) and deconvolving it from the measured voltage.

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