

Non-dimensional Performance Trends of a Pulsed Plasma Accelerator

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The non-dimensional performance trends of a pulsed plasma accelerator that is prone to mass leakage out of the current sheet and into a wake are presented. The non-dimensional total impulse, specific impulse and efficiency are found to depend in a straightforward manner on the non-dimensional current sheet and wake mass and velocity, which are experimentally measured. The performance is found to depend on the propellant species and pressure. Specifically, performance increases with pressure for hydrogen and helium propellants, stays constant for neon and decreases for argon. These trends indicate that the leakage process, which differs for different propellants, has a large influence on the performance of the device as a plasma accelerator.

Nomenclature

| | |
|-------------|---|
| \bar{c} | Ion thermal velocity, m/s |
| E_0 | Initial energy, J |
| g_0 | Acceleration due to gravity, m/s ² |
| I_{sp} | Specific impulse, s |
| I_{total} | Total impulse, Ns |
| K | Kinetic energy, J |
| m | Mass, kg |
| n | Number density, m ⁻³ |
| v | Velocity, m/s |
| v^* | Characteristic velocity, m/s |

Subscripts

| | |
|------|-----------|
| a | Ambient |
| av | Available |
| c | Cathode |
| e | Electron |
| i | Ion |
| sh | Sheet |
| w | Wake |

Symbols

| | |
|------------|--|
| Γ^e | Flux of mass entering the sheet, kg/s |
| Γ^i | Flux of mass leaking from the sheet at the cathode, kg/s |

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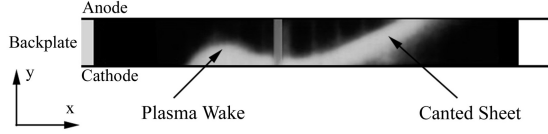


Figure 1. Photograph of a discharge from Ref. 4, showing two non-idealities of current sheet behavior. The top electrode is the anode and the bottom electrode is the cathode. The plasma is moving from left to right. The ideal behavior would be for the current sheet to be perpendicular to the electrodes. Instead we see a canted sheet and a plasma wake trailing the sheet along the cathode. The vertical bar in the middle of the photograph is a physical structure that obscured the light, not a plasma structure.

| | |
|------------|-------------------------------------|
| η | Efficiency |
| θ | Canting angle of the sheet, degrees |
| ξ | Non-dimensional velocity |
| ρ | Density, kg/m ³ |
| Φ | Non-dimensional mass |
| Ω_i | Ion Hall parameter |

I. Introduction

THE desired behavior of the current sheet in a gas-fed pulsed plasma thruster with a uniform gas fill is that it follows the “snowplow model” of development. The snowplow model describes a progression of the sheet from an initiation phase to a sweeping phase and finally expulsion.¹ Since higher efficiency of operation translates directly into reduced mass and cost of propellant, it is important to identify and quantify phenomena of the discharge that effect the performance of the device. Two major non-idealities of current sheet behavior have been identified, current sheet canting^{2,3} and current sheet mass leakage.⁴ The tendency of mass to leak from the current sheet into a wake is explored in the present study.

Previous research has shown that the operation of these devices is not likely to follow the ideal pattern of the snowplow model. Recently, a study by Markusic⁵ at Princeton University’s Electric Propulsion and Plasma Dynamics Laboratory showed that the behavior of current sheets was more complicated. Markusic’s study focused on the tendency of the current sheet to cant, or tilt, with the anode attachment leading the cathode attachment. This behavior was evident under all conditions tested, and is shown in Fig. 1. Also evident from Markusic’s study was that the current sheet seemed to be leaking plasma along the cathode as it traveled. This is also visible in figure 1. The present study focuses on this phenomenon of current sheet leakage. It is clear that leakage of the propellant through the current sheet could lead to a loss of momentum of that portion of the propellant. This can have a negative effect on the performance of the device. Although researchers have seen current sheet leakage in the past,^{2,3,5–13} a study of its effect on the performance of the accelerator has never been performed. Also, knowledge of the fundamental physics behind the leakage phenomenon and the partitioning of momentum into the sheet and wake was lacking. It is these questions which the current study will investigate, and hopes to answer.

The goal of the present study is to explore the performance trends of a gas-fed pulsed plasma accelerator. This is accomplished by making experimental measurements of the current sheet and wake mass and velocity, and using these measurements to determine the non-dimensional impulse and efficiency of the device.

In the next section, the non-dimensional performance indicators of a pulsed plasma accelerator are introduced and we explain how the current sheet mass leakage problem could effect the performance. The non-dimensional sheet and wake mass measurements are then presented, followed by the non-dimensional sheet and wake velocities. These quantities are used to calculate the non-dimensional impulse and efficiency and finally a discussion of the experimental trends is presented.

II. Apparatus

The device used in this study has been previously described in detail.² Only a brief description will be provided here. The accelerator is not an actual thruster, but rather an idealized parallel plate accelerator with Pyrex sidewalls that allow good optical access to the discharge. The anode and cathode are made of

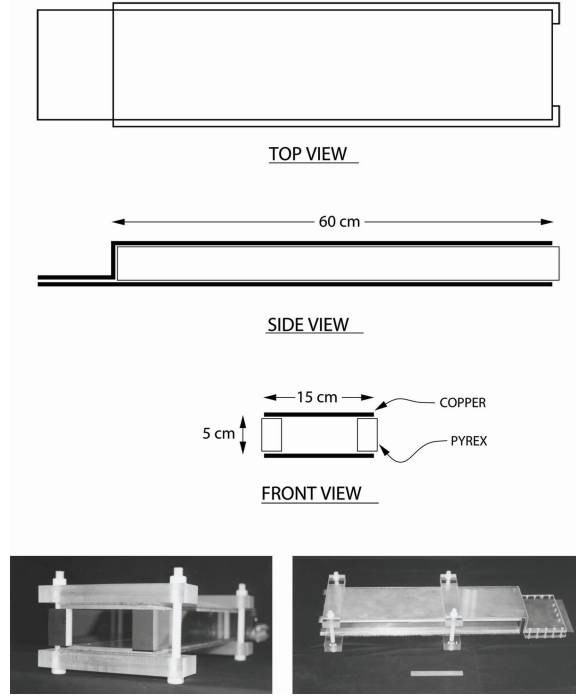


Figure 2. Schematic and photographs of the parallel plate accelerator.

copper and the volume of the acceleration region measures 60 cm long and 10 cm wide, with a gap of 5 cm between the plates. The initial propellant distribution is a uniform gas fill. A pulse forming network is used to supply a current pulse with an approximately constant current of ~ 60 kA for $25 \mu\text{s}$, when charged to 9 kV. Figure 2 shows a schematic and photographs of the accelerator.

III. The Effect of Mass Leakage on Accelerator Performance

York and Jahn introduced a “sweeping parameter” which is the percent of available mass contained in the sheet.¹⁰ This parameter is defined,

$$\Phi_{sh} = \frac{m_{sh}}{m_{av}}, \quad (1)$$

where m_{sh} is the mass of the sheet, and m_{av} is the total available mass. In the experiments presented here, the accelerator is operated with a uniform gas fill, so the available mass is simply the density of the gas fill times the volume of the accelerator.

The sweeping parameter, or non-dimensional current sheet mass, is interesting to know in its own right, but further motivation for this study can be drawn from quantifying its effect on the thruster performance. There are three important performance measurements of a pulsed plasma accelerator: total impulse, specific impulse, and efficiency.

Total impulse, the analog of thrust in a steady-state device, is the momentum delivered by a single pulse of the device. This includes the momentum of the current sheet and the plasma wake. We will make the assumption that the available mass is either contained in the sheet or the wake, or it is lost entirely. The third possibility would happen, for example, if neutral particles were to permeate through the sheet without gaining momentum. “Permeability” of the current sheet refers to the tendency of ambient neutral particles to slip through the sheet and be left behind, and it should be distinguished from “leakage” of the current sheet, which refers to the systematic loss of mass from the sheet to the wake at the cathode.

If we also make the assumptions that the particles in the sheet and wake travel with mass-averaged velocities, then the total impulse can be written, in terms of the sweeping parameter, as:

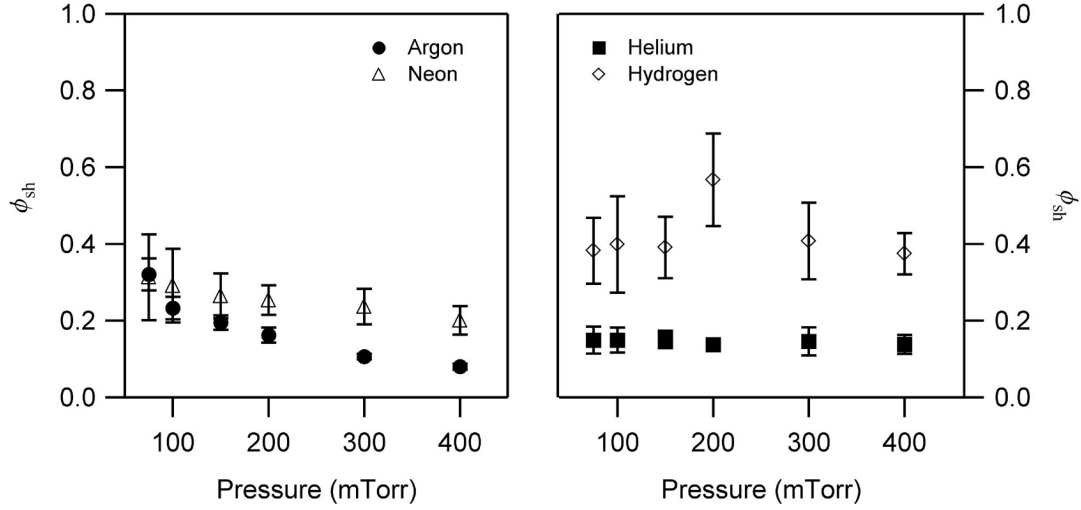


Figure 3. Non-dimensional current sheet mass vs. pressure for argon and neon, in the left panel and helium and hydrogen on the right.

$$I_{total} = m_{sh}v_{sh} + m_wv_w = m_{av}(\Phi_{sh}v_{sh} + \Phi_wv_w). \quad (2)$$

The sheet mass and velocity are represented by m_{sh} and v_{sh} and the wake mass and velocity are indicated by m_w and v_w . We have also introduced a parameter, Φ_w , for the wake that is analogous to the sweeping parameter of the sheet.

The velocity of the sheet is greater than the velocity of the wake, so it is immediately obvious, and intuitive, that a higher sweeping parameter (a larger percentage of the available mass is contained in the sheet) should lead to a higher total impulse.

We can further non-dimensionalize the total impulse equation by defining a characteristic velocity. Since the characteristic mass used was m_{av} a logical characteristic velocity to use is,

$$v^* = \sqrt{\frac{2E_0}{m_{av}}}, \quad (3)$$

where E_0 is the initial energy stored in the pulse forming network. This is what the velocity would be if all the energy put into the discharge was converted into kinetic energy, and it accelerated all of the available mass. Of course, the velocities achieved in our experiments are merely fractions of this velocity (10% - 25%).

Now, we can write the non-dimensional total impulse as,

$$\hat{I}_{total} = \Phi_{sh}\xi_{sh} + \Phi_w\xi_w, \quad (4)$$

where $\xi_{sh} = v_{sh}/v^*$ and $\xi_w = v_w/v^*$.

Also, we can write the specific impulse and efficiency based on the same non-dimensional parameters. The specific impulse of a thruster is the total impulse divided by the propellant weight,

$$I_{sp} = \frac{I_{total}}{m_{av}g_0}, \quad (5)$$

$$\hat{I}_{sp} = \frac{I_{sp}}{v^*/g_0} = \Phi_{sh}\xi_{sh} + \Phi_w\xi_w. \quad (6)$$

We find that the non-dimensional specific impulse is the same as the non-dimensional total impulse. This is not surprising since we have non-dimensionalized the total impulse with the available mass, which gives it the same meaning as the non-dimensional specific impulse.

Next we consider the efficiency of the device. The acceleration efficiency is the percentage of input energy that is converted into kinetic energy. Efficiency is already non-dimensional:

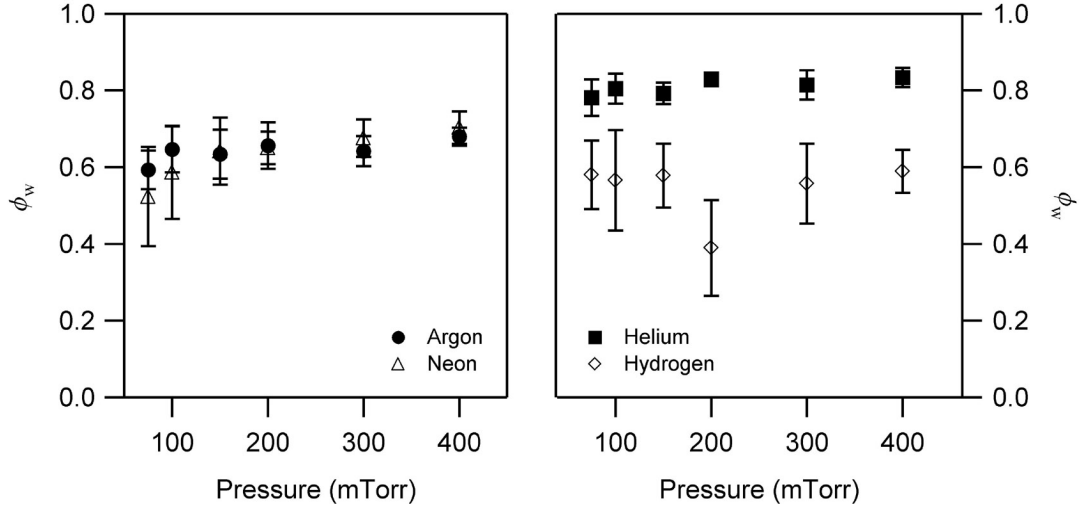


Figure 4. Non-dimensional wake mass vs. pressure for argon and neon, in the left panel and helium and hydrogen on the right.

$$\eta = \frac{K_{sh} + K_w}{E_0} = \Phi_{sh}\xi_{sh}^2 + \Phi_w\xi_w^2. \quad (7)$$

Again we see that, since $\xi_{sh} > \xi_w$, the efficiency increases with increasing sweeping parameter.

It is clear that four parameters must be measured to characterize the performance of the device: the mass and velocity of the sheet and the mass and velocity of the wake. The goal of this work is to explore the performance of the accelerator under different operating conditions and to determine how the mass leakage process effects the performance. We will thus characterize the dependencies of the four non-dimensional parameters, Φ_{sh} , Φ_w , ξ_{sh} , and ξ_w on experimental conditions.

IV. Non-Dimensional Masses

The first parameter is the non-dimensional current sheet mass, or the sweeping parameter. The current sheet mass was measured by making time-resolved measurements of the current sheet electron number density with a laser interferometer and then integrating across the sheet volume.

First, we assume that the plasma in the sheet is singly ionized, therefore the electron number density is equal to the ion number density. Second, we can assume that the sheet is fully ionized, so that ions make up the entire mass of the sheet. The total mass of the sheet, then, is the density integrated over the volume,

$$m_{sh} = \int \rho_i dV = \int \int \int m_i n_e dx dy dz. \quad (8)$$

In our experiment, the sheet is assumed uniform in the z direction (out of the page in figure 1) therefore the dz term immediately comes out of the integral as d , the “depth” of the electrodes. Integrating the density across the sheet width in the x direction is equivalent to integrating density times velocity with respect to time. The velocity of the sheet is also measured. In the y direction, the plasma density is not uniform, however we have consistently measured a linear increase in density from anode to cathode. With this assumption, only a density measurement in the middle of the electrode gap is necessary to determine the sheet mass. This method of measuring the current sheet mass from interferometry is described in much more detail in Refs. 4 and 14.

We can then obtain a measure of the sweeping efficiency from our sheet mass measurements by dividing by the available propellant mass, according to Eq. (1). Figure 3 shows the sweeping efficiency of discharges in various propellants vs. pressure. For the heavier propellants Φ_{sh} decreases with increasing propellant fill pressure. This is another way of expressing that the sheet mass does not increase at a rate commensurate with the increase in propellant pressure. For helium, the sweeping efficiency stays constant with pressure,

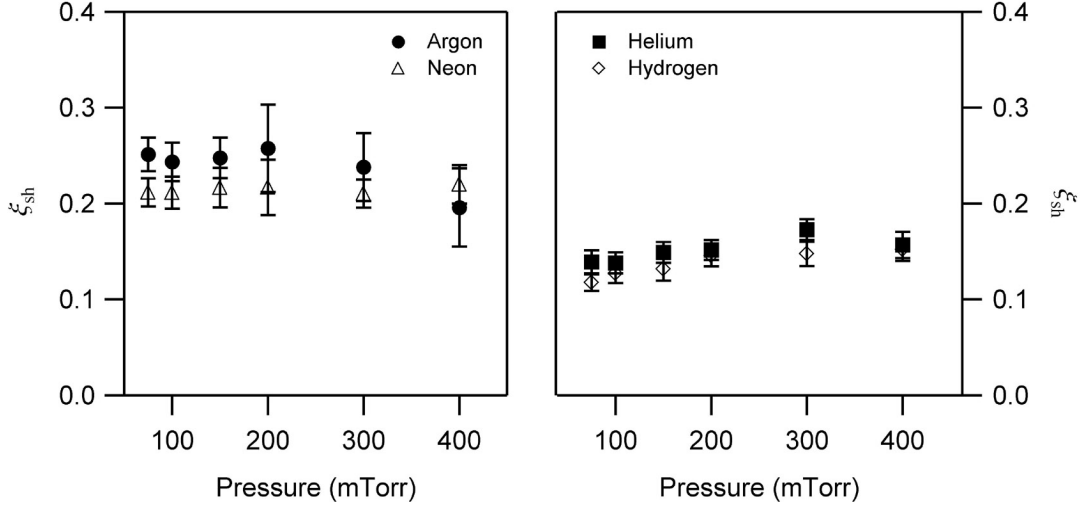


Figure 5. Non-dimensional current sheet velocity vs. pressure for argon and neon, in the left panel and helium and hydrogen on the right.

but it is quite low, around 20%. For hydrogen, Φ_{sh} also is constant with pressure at around 40%, with the exception of the 200 mTorr measurement. We believe that this single measurement is anomalously high and not indicative of a trend.

Next we present the analogous parameter for the wake instead of the sheet. This is the percent of available mass that goes into the wake, $\Phi_w = m_w/m_{av}$ (Fig. 4). In this case, measuring the wake mass directly through interferometry is not possible because of the unknown volume and distribution of density in the wake. Instead, the wake mass has been inferred from measurements of the sheet mass and the mass left behind in the accelerator and the assumption that the available propellant mass is divided into these three possible outcomes. The left-behind mass was measured directly by measuring the mass of a “restrike” current sheet that forms in the accelerator when the current reverses after about 25 μ s. This second sheet has been observed to sweep up all of the mass left behind by the sheet and the wake.⁴ The inferred wake mass measurements have been validated, at least to first order, by momentum plate measurements of the total impulse which agreed reasonably with the added-up impulse of all the measured components.⁴

Hydrogen and helium discharges have very small observed restrike and current sheet masses, implying that all the available mass is contained in the sheet and wake and there is no permeability of propellant through the sheet. Therefore the non-dimensional wake mass is approximately given by $\Phi_w = 1 - \Phi_{sh}$. For argon and neon, however, $\Phi_{sh} + \Phi_w < 1$, because of mass that is lost from both structures due to permeability of the sheet.

V. Non-Dimensional Velocities

The third and fourth parameters on which the performance of the device depends are the non-dimensional current sheet and wake velocities. In section III we defined these parameters by dividing them by the velocity that would be obtained if all of the input energy went into kinetic energy. Thus, the non-dimensional velocities are defined as,

$$\xi_{sh} = v_{sh} \sqrt{\frac{m_{av}}{2E_0}}, \quad (9)$$

$$\xi_w = v_w \sqrt{\frac{m_{av}}{2E_0}}. \quad (10)$$

Using the input energy (which is 4050 J for all 9 kV discharges), the available propellant mass and the measured sheet and wake velocities, we can calculate these non-dimensional velocities. The velocity measurements were made by using high-speed photography to track the motion of the sheet and wake from

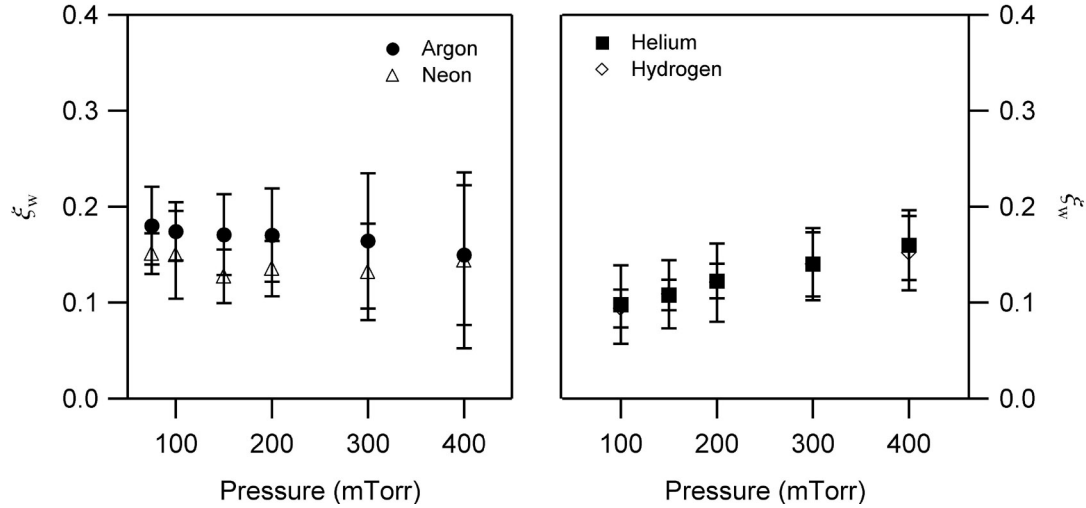


Figure 6. Non-dimensional wake velocity vs. pressure for argon and neon, in the left panel and helium and hydrogen on the right.

frame to frame.^{4,14} Between four and twelve exposures were made per measurement condition, with an inter-frame time of $0.5 \mu\text{s}$.

Figure 5 shows ξ_{sh} vs. pressure, while Fig. 6 shows ξ_w vs. pressure. We can see from these figures that the non-dimensional sheet and wake velocity are both fairly constant with pressure for argon and neon, except for argon sheet velocity at the highest pressure, which begins to drop off. For helium and hydrogen, both non-dimensional velocities increase with pressure, the wake velocity at a slightly higher rate than the sheet velocity.

VI. Non-Dimensional Impulse

In section III we identified three major performance indicators of pulsed plasma thrusters: total impulse, specific impulse and acceleration efficiency. We found that these three indicators depend upon the four non-dimensional parameters presented above: non-dimensional sheet mass, wake mass, sheet velocity and wake velocity. Specifically, the non-dimensional impulses are calculated from Eq. (4), and the efficiency is calculated from Eq. (7).

Figure 7 shows the measured non-dimensional impulses calculated from the four measured non-dimensional parameters. The non-dimensional impulse decreases with increasing pressure for argon, stays constant for neon and increases for helium and hydrogen. Because we have written the non-dimensional impulse as a function of the four measured non-dimensional parameters, it is easy to see how the interplay of these parameters cause the trends in the performance seen here. The decreasing performance with increasing pressure for argon is clearly due to the fact that the non-dimensional sheet mass, Φ_{sh} , decreases with pressure at a rate higher than that of the increase in the non-dimensional wake mass, Φ_w . For neon, the two rates are nearly equal and opposite, ensuring that impulse lost from the sheet at higher operating pressures is made up for by increased wake impulse. Thus the non-dimensional impulse stays constant with pressure for neon. For helium and hydrogen the Φ 's are both constant with pressure. The slight increase in performance with pressure, then, is due to the increase in the non-dimensional velocities. Another interesting trend is that the non-dimensional impulse is nearly equal for the helium and hydrogen current sheets. This is despite the fact that the division of mass between the sheet and wake differs for these propellants by about a factor of two (hydrogen sheets contained about 40% of the available mass compared to about 20% for helium). The reason for this is that the wake velocities are actually quite close to the sheet velocities for these propellants, so that there is not much loss of impulse by transferring mass from the sheet to the wake.

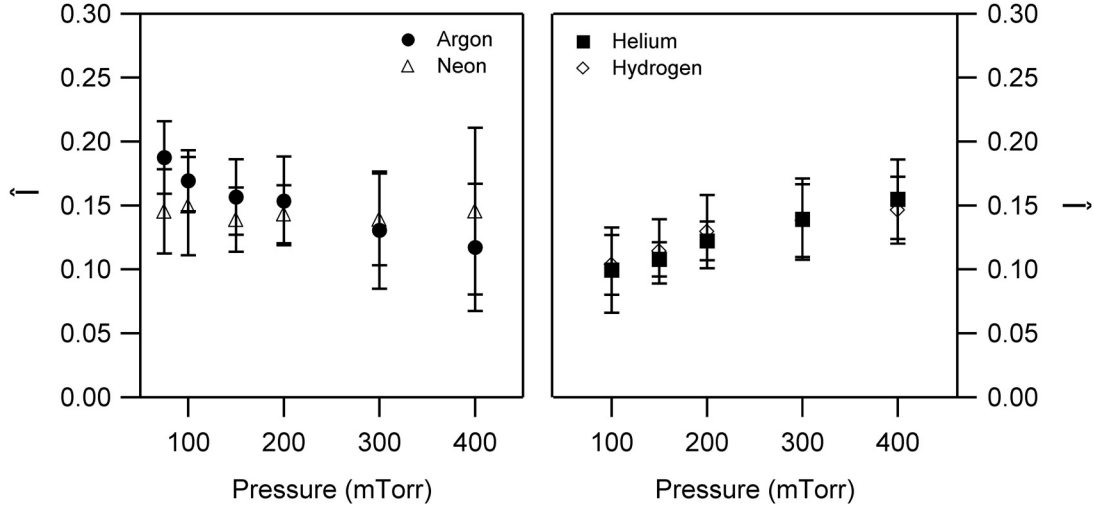


Figure 7. Non-dimensional impulse versus pressure for argon and neon, in the left panel and helium and hydrogen on the right.

VII. Efficiency

Another measure of the performance of the device is acceleration efficiency, which is the kinetic energy of the sheet and wake exhaust divided by the input energy. The efficiency is calculated from Eq. (7). Figure 8 shows the efficiency versus pressure for each propellant. The trends that are evident here are the same as those from the non-dimensional impulse plots above. The performance decreases with pressure for argon, is approximately constant with pressure for neon and increases for helium and hydrogen. In this case, the trends are even further exaggerated, especially for helium and hydrogen, due to the effect of the extra velocity terms in Eq. (7).

VIII. Discussion

In this paper we have presented measurements of the current sheet and wake mass and velocity in a parallel plate pulsed plasma accelerator. These measurements allow calculations of the performance parameters non-dimensional impulse and efficiency. Laser interferometry was used to obtain a time-resolved electron number density measurement in the sheet, which was in turn used to calculate the mass of the current sheet. The wake mass was inferred from the sheet mass measurements and measurements of the restrike current sheet mass, which was assumed to contain all of the mass left behind by the two structures. The sheet and wake velocities were measured straightforwardly by high-speed photography. Each of the four parameters of sheet and wake mass and velocity were non-dimensionalized for easy comparison between propellants.

Two key performance parameters were calculated from the measured quantities above. These were the non-dimensional impulse and the efficiency. Through these calculations, major performance trends have been identified: thrusters operating with argon propellant have decreasing performance with increasing propellant pressure, with neon performance is constant and with helium and hydrogen performance increases. Because of the simple relationship of the performance parameters on the four measured non-dimensional quantities, it is straightforward to identify what causes these different trends.

- For argon, performance decreases with pressure. This is because Φ_{sh} decreases with pressure and Φ_w does not increase sufficiently to make up for the loss. In other words at high pressures there is a greater loss of mass from both structures.
- For neon, performance is constant with pressure. In this case the loss of mass from the sheet is made up for by an increase in the wake mass.
- For helium and hydrogen, performance increases with pressure. Since both Φ_{sh} and Φ_w are constant with pressure, the performance increase comes from the increase in ξ_{sh} and especially ξ_w .

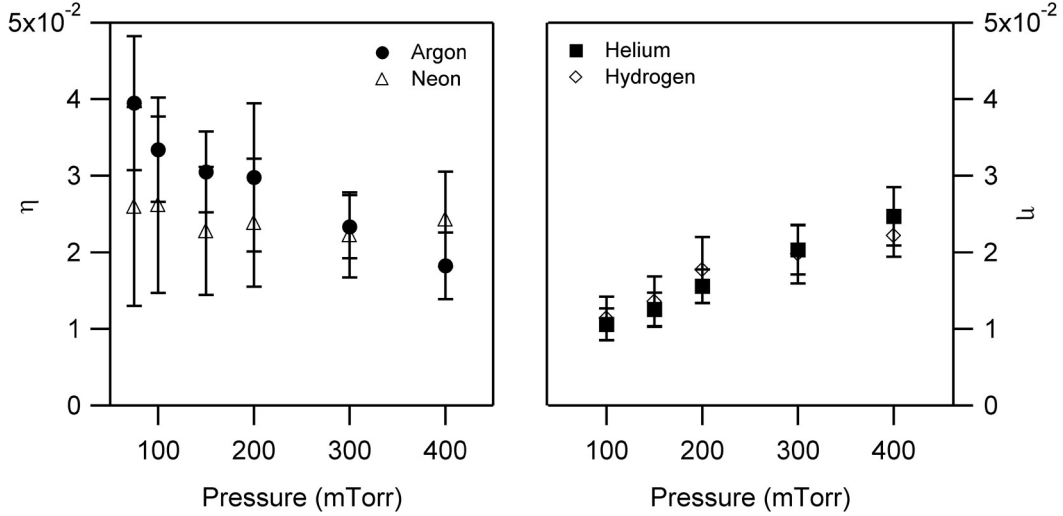


Figure 8. Efficiency versus pressure for argon and neon, in the left panel and helium and hydrogen on the right.

A. Analytical Model

The trends presented here indicate that the leakage process, which differs for different propellants, has a large influence on the performance of the device as a plasma accelerator. It is clear that the trend of the sweeping efficiency, in particular, plays a large role in determining the performance trends of the accelerator. A model of the current sheet has been developed to address the issue of the trend of the sweeping efficiency (as shown in Fig. 3) and its dependence on propellant species.^{4,14}

Much more detail is included in references 4 and 14, but a brief description of the model will be provided here as well. In the steady-state propagation phase of the discharge the sheet mass is constant and therefore a balance exists between the flux of mass into the sheet and the leakage of mass out of the sheet. This can be expressed as

$$\frac{\partial m_{sh}}{\partial t} = 0 = \Gamma^e - \Gamma^l, \quad (11)$$

where Γ^e is the flux of mass entering the sheet, and Γ^l is the flux of mass leaking from the sheet at the cathode and entering the wake. Since $\Gamma^e \propto \rho_a v_{sh}$ and $\Gamma^l \propto \rho_i v_c$, we can write:

$$\Phi_{sh} \propto \frac{\rho_i}{\rho_a} \propto \frac{v_{sh}}{v_c}, \quad (12)$$

where ρ_a is the ambient neutral propellant density, and v_c is the velocity of sheet ions towards the cathode.

It was found that in the lighter propellants, which have a higher ion Hall parameter ($\Omega_i \gtrsim 1$), the ions in the sheet are able to complete a drift motion perpendicular to the current sheet. In this case $v_c = v_{sh} \tan \theta$, where θ is the canting angle, and from Eq. (12), Φ_{sh} is constant with pressure. This is what is observed in Fig. 3.

For the heavier propellants, which have low ion Hall parameters ($\Omega_i \ll 1$), ions are not able to complete drift motions and instead move only in the x direction with the sheet velocity. This would imply that the velocity towards the cathode is zero and the sheet continues to gain mass, which is not the case. Instead, the leakage mechanism in this case can be explained by a diffusive flux of ions to the cathode, such that $v_c = \bar{c}/4$. In this case we see, from Eq. (12) that $\Phi_{sh} \propto v_{sh}$. For argon and neon propellants, we can see in Fig. 3 that the sweeping efficiency decreases with pressure with the same trend as v_{sh} . Therefore the difference in the trends of Φ_{sh} with pressure between the heavy and light propellants can be explained by a difference in ion Hall parameters.

Acknowledgments

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