

Design of an Experiment to Optimize Plasma Energization by Beating Electrostatic Waves

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The design and implementation of an experiment dedicated to testing the wave frequency and amplitude dependence of the heating of a magnetized plasma with beating electrostatic waves is discussed. This non-resonant heating process has the potential to be more efficient than current radio frequency plasma heating methods and thus is particularly promising for electrothermal plasma propulsion. Recent theoretical work on beating electrostatic wave heating is reviewed with an emphasis on the existence of an optimal frequency for maximum heating as well as a scaling relation for the magnitude of heating. While the possibility of optimizing the process thus is demonstrated theoretically, systematic experimental verification remains to be conducted. The implementation of a cylindrical, magnetized plasma generated with an inductive source is discussed as a testbed for the predicted theoretical trends. The diagnostics for observing the amplitude and frequency dependence are also presented as well as a derivation of the experimental parameters necessary for the tests to accurately investigate the theoretical predictions.

I. Introduction

The Radio Frequency (RF) heating of plasma is an essential process in a number of industrial and scientific processes and in recent years has seen an increasing use in electrothermal plasma propulsion. RF heating in this application is particularly attractive as it is electrodeless (and thus extends thruster lifetime) and potentially very efficient.¹ Typical RF heating schemes rely on resonances between the exciting waves and particle dynamics. In Ion Cyclotron Resonance Heating (ICRH), for example, a small fraction of the magnetized ions that are resonant with the exciting waves are subject to energy exchange with the wave. The remainder of the ion population is subsequently energized through the secondary effect of collisional processes. By contrast, in a non-resonant heating process, the *entire* initial distribution of ions interacts with the waves, and as a consequence, this process is thought to be theoretically more efficient than resonant RF heating.² It is thus with considerable interest that researchers have investigated non-resonant forms of ion acceleration. Most notably, Benisti *et.al* demonstrated theoretically that non-resonant acceleration can be achieved through the non-linear interaction of magnetized ions with two electrostatic waves propagating perpendicularly to the magnetic field and subject to the beat criterion, $\omega_1 - \omega_2 = n\omega_c$ where ω_1 and ω_2 are the frequencies of the exciting waves, ω_c is the cyclotron frequency of the ions, and n is a positive integer.³

While the process Benisti *et.al* reported was non-resonant in that ions with arbitrarily low initial energy were accelerated in the presence of Beating Electrostatic Waves (BEW), it was apparent that the effect did not accelerate ions for all possible initial conditions. Choueiri and Spektor subsequently examined the case of $n = 1$ and showed that there are necessary and sufficient conditions on the initial Hamiltonians of the ions (in addition to the beat criterion) in order for acceleration to occur.⁴ They further demonstrated through numerical simulations that even when taking the constraints for acceleration into account, a physical, collisionless plasma should experience a net increase in energy. We have recently expanded on this work by analytically showing that BEW heating does produce an increase in average ion speed in an ensemble and moreover that the increase depends directly on the amplitude and the frequency of excitation of the waves.⁵ In particular, we have demonstrated that for a given initial temperature of the plasma, there is an optimal frequency that produces the largest increase in average ion speed in the ensemble. We further have derived

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a scaling relation for how the magnitude of heating depends on wave amplitude. The characterization of these trends is a particularly important development for electrothermal plasma propulsion as they represent an initial step toward optimizing the BEW heating process.

While Spektor and Choueiri⁶ have successfully demonstrated experimentally that BEW heating does in fact produce an increase in ion temperature, due to difficulties in wave launching and the limited diagnostics of their experiment, they were unable to explore the frequency and amplitude dependence of the magnitude of heating. The need is apparent then for an experimental testbed with which we can determine if an optimal frequency does in fact exist and how the wave amplitude influences heating. With this end in mind, it is the goal of this paper to outline the design and implementation of an experiment that unambiguously characterizes the frequency and amplitude dependence of BEW heating. In Section II, we provide a review of BEW single ion acceleration as well as the derivation of the frequency and amplitude dependence of BEW plasma heating. In Section III, we outline the primary and secondary objectives of our experiment. In Section IV, we describe the experimental apparatus. And finally, in Section V, we discuss the direction of future investigations.

II. Review of Theoretical Findings

A. Single Particle Dynamics

The theoretical understanding of plasma heating with BEW evolved from an examination of the acceleration of an individual ion by multiple electrostatic waves. The geometry of the ion motion is depicted in Figure 1 where we take the magnetic field to be constant in the \hat{z} direction and the electrostatic waves to propagate in the \hat{x} direction. In the unperturbed case (no incident electrostatic waves), the ion has tangential velocity v_\perp

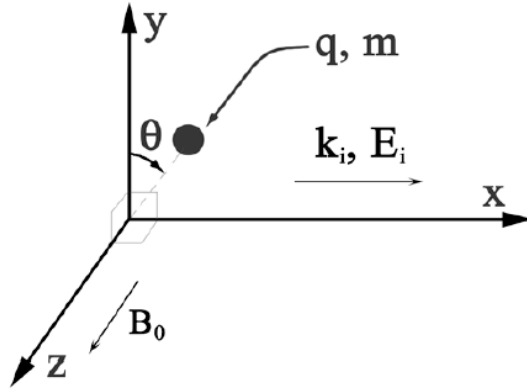


Figure 1. A single ion of charge q and mass m in a constant homogeneous magnetic field $B\hat{z}$ interacts with an electrostatic wave. The wavenumber and electric field of the wave are parallel to the x axis.

and undergoes simple Larmor precession at the cyclotron frequency ω_c with Larmor radius given by r_L . In the presence of two electrostatic waves propagating perpendicularly to the magnetic field, the ion dynamics become significantly more complicated. In particular, the equation of motion in the Cartesian formulation is given by

$$\ddot{x} + \omega_c^2 x = \sum_{i=1,2} E_i \sin(k_i x - \omega_i t), \quad (1)$$

where E_i and k_i are the amplitude and wave number respectively of the i^{th} wave. Since this non-linear differential equation is intractable, we must resort to numerical and perturbation methods in order to understand its behavior. To this end, the equation is more readily analyzed in a normalized, action-angle formulation where the Hamiltonian for the equations of motion is given by^{3,4}

$$H = I + \sum_{i=1,2} \varepsilon_i \cos(\sqrt{2I} \sin \kappa \theta - \nu_i \tau + \varphi_i), \quad (2)$$

where H is the Hamiltonian of the system, $\nu_i = \omega_i/\omega_c$, $\tau = \omega_c t$, $\varepsilon_i = (kqE_i)/(m\omega_c^2)$, $k = k_1$, $\kappa = k_i/k$ and $\bar{2I} = \rho = r_L/k$. Here q and m are the charge and mass of the ion respectively, ρ is the normalized Larmor radius, φ_i is the phase of each wave relative to the cyclotron motion, and θ is the cyclotron rotation angle measured from the \hat{y} direction. With this formulation, Benisti and Ram^{3,7} were able to show that the maximum acceleration for any ion occurs when the waves have equal amplitude, $\varepsilon_1 = \varepsilon_2 = \varepsilon$, and equal wavenumber, $k_1 = k_2 = k$. We thus restrict our analysis to these conditions, and for simplicity we follow the precedent set by Choueiri and Spektor⁴ in considering the case of $n = 1$ such that $\nu = \nu_1 = \nu_2 = 1$ and where the waves are in phase such that $\varphi_1 = \varphi_2 = 0$. The Hamiltonian governing the equation of motion then is given in a simplified form by

$$H = I + \varepsilon \left(\cos(\bar{2I} \sin \theta - \nu \tau) + \cos(\bar{2I} \sin \theta - (\nu + 1)\tau) \right). \quad (3)$$

B. Plasma Heating with BEW

The derivation of Eq. 3 is strictly for the case of single-particle motion. In order to characterize ion dynamics in an actual plasma, an ensemble of ions must be considered, and additional terms representing collective effects and collisionality must be included. However, as has been reported in Ref. 5, these additional terms can be ignored, and Eq. 3 can accurately be used to describe motion in a plasma if three criteria are satisfied. First, the heating time scale t_h is shorter than the time between de-phasing collisions t_i such that $t_h \ll t_i$. De-phasing collisions consist of any ion-particle interactions that result in a change in phase angle. As demonstrated by Spektor and Choueiri,² these are the types of collisions that render additional terms in Eq. 3 necessary while by contrast non-dephasing collisions allow the ions to continue on as if the equations of motion have been unchanged. Second, the exciting waves are assumed to be independent of the ion dynamics. Third, we explore only the non-resonant case for each wave such that $\omega_1, \omega_2 = m\omega_c$ where m is a positive integer. Assuming these three criteria are satisfied, we are free to use Eq. 3 in a discussion of the macroscopic variables of an ion ensemble. Specifically, as outlined in Ref. 5, it is desirable to know how the average speed of the ions in the plasma will evolve in the presence of BEW as this gives an indication of the magnitude of heating. Since $\rho = \bar{2I}$ scales directly with the ion speed, this variable is the focus of our investigation.

If the equations of motion described by Eq. 3 are tractable, then for a given initial action I_0 and Larmor angle θ_0 , we can find the value of the action I at some subsequent time τ : $I(\tau) = I(\theta_0, I_0, \tau)$. Furthermore, if we know the initial distribution function of the ions, $f(\theta_0, I_0)$, we can find the average value of any function of $I(\tau)$ at time τ . In particular, for $\rho = \bar{2I}$

$$\rho(\tau) = \int_0^\infty \int_0^{2\pi} \sqrt{2I_0} f(\theta_0, I_0) d\theta_0 dI_0, \quad (4)$$

where the integrals are performed over all possible values of initial Larmor angle and action. We further posit that the plasma is initially in thermal equilibrium such that the the distribution of ions in phase space is Maxwellian. We also assume that before heating, the collisionality is sufficiently high that the initial Larmor angles are randomized. In action-angle coordinates then, the initial distribution function is strictly a function of the action I_0 and is given by

$$f(I_0) = \frac{4}{\pi} \beta^{3/2} \sqrt{2I_0} e^{-\beta^2 I_0}, \quad (5)$$

where k_B is the Boltzmann constant, T is the temperature of the ion distribution in Kelvin, and $\beta = \left(\frac{m}{2\pi k_B T} \right) \left(\frac{\omega_c}{k} \right)^2$. β is a direct measure of the spread in the Maxwellian distribution, and physically, β scales with the square of the ratio of the thermal Larmor radius of ions in the plasma ($r_{L(th)} = v_{th}/\omega_c = 8k_B T / \pi m \omega_c$ where v_{th} is the thermal velocity of ions) to the wavelength of the exciting waves ($\lambda = 2\pi/k$) such that $\beta = (\lambda/r_{L(th)}\pi^2)^2$. For optimal coupling, λ/r_L is on the order of unity. Therefore, typical values of β will range from 10^{-2} to 10^{-1} .

With $f(I_0)$ strictly a function of the action, substitution into Eq. 4 yields

$$\rho(\tau) = \int_0^\infty \left\langle \sqrt{2I(I_0, \theta_0, \tau)} \right\rangle_{\theta_0} f(I_0) dI_0, \quad (6)$$

where we have integrated over the initial action and denoted for subsequent convenience the average over initial phase angle as $\dots \theta_0$. Eq. 6 is easily evaluated if an analytical solution exists for Eq. 3. However, this Hamiltonian is non-linear, and there is no closed form solution for $\sqrt{2I(I_0, \theta_0, \tau)}$. It is therefore necessary to employ a perturbation method. Specifically, for Hamiltonians of the form of Eq. 3, there exists a second-order, phase-averaged, Lie Transform method for finding an approximation for phase angle-averaged quantities.^{8,9} Using this method as we did in Ref. 5, we can evaluate Eq. 6 in the the small $\varepsilon < 1$ limit to second-order:

$$\rho(\tau)_2 = \sum_{i=1,2;j=1,2} \varepsilon^2 \sum_{-\infty}^{\infty} \int_0^{\infty} \frac{\partial}{\partial I_0} \left[\frac{m^2 J_m(\frac{2I_0}{\beta})^2}{2I_0} \right] f(I_0) dI_0 \quad (7)$$

$$\frac{1}{2(\nu_i - m)(\nu_j - m)} [\cos([\nu_i - \nu_j] \tau) + \cos([m - \nu_i] \tau) + \cos([m - \nu_j] \tau) + 1] + \frac{2}{\pi\beta},$$

where the summation is over integer values of m , and J_m is the Bessel function of the first kind. The above expression represents the time evolution of the average ion speed, and it is immediately noted that it exhibits periodicity in time. This implies that after averaging over several multiples of the longest period term, the average normalized ion speed will equilibrate. Following our work from Ref. 5 in which the small β assumption is employed and only resonant terms retained, we average the expression and find a simplified result for the equilibrated value of the average ion speed in the plasma when subject to BEW:

$$\rho_{2\text{eq}} = \frac{2}{\pi\beta} + \left(\frac{\varepsilon}{\delta}\right)^2 \frac{4}{\pi} \beta^{3/2} \left((\nu - \delta)^2 \sqrt{\frac{\beta}{\pi}} e^{-\beta(\nu-\delta)^2} + \frac{\nu}{2} \frac{\delta}{2} \left[\text{Erf}\left(\frac{(\nu - \delta)\sqrt{\beta}}{1}\right) - 1 \right] \right), \quad (8)$$

where $\text{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2} dx$ is the error function and $\delta = \min |\nu - m|$.

This result provides a concise depiction of how the equilibrated average ion speed in an initially Maxwellian plasma depends on the amplitude of the waves ε , the spread in the Maxwellian β , and the frequency of excitation ν . A sample depiction of Eq. 8 for fixed values of β and ε and varying normalized frequency ν is shown in Figure 2. It is immediately apparent that not only does the average speed of the ions experience an increase, there is an optimal frequency at which the maximum increase in ion average ion speed occurs. This is a particularly exciting observation as it implies it may be possible to optimize the beating wave heating process. Furthermore, the magnitude of the heating is seen to scale with the amplitude of the exciting waves—offering another method by which the BEW mechanism may be controlled. We can find expressions for how this scaling relation as well as the optimal frequency depend on the experiment and wave parameters.

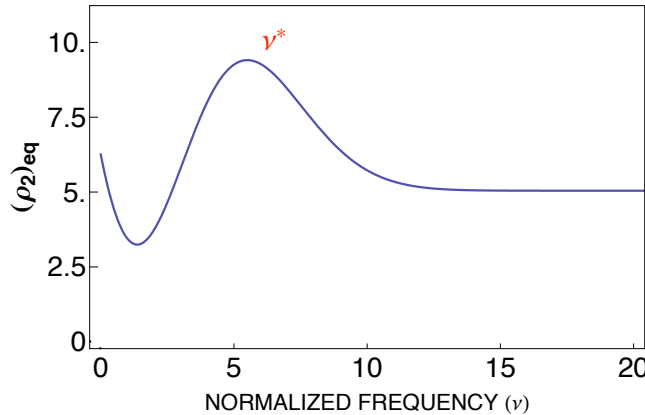


Figure 2. Second order prediction for the frequency dependence of the equilibrated value of average ion speed. $\beta = .05$ and $\varepsilon = 1$.

1. Optimal frequency for plasma heating

The value of the optimal frequency depicted in Figure 2 is denoted ν^* and occurs where the the average ion speed has the largest equilibrated value. We have found this maximum persists for a wide range of β and

ε , and we have provided a physical justification for its existence in Ref. 5. We also have noted that in the small β limit, the value for the optimal frequency is given by

$$\nu^* = \sqrt{\frac{3}{2\beta}} + \delta. \quad (9)$$

We checked the validity of this expression numerically in Ref. 5 and found it to be accurate for the $\beta < 0.1$ and $\varepsilon < 1$ limits.

2. Magnitude of plasma heating

For the small ε limit, we find at a fixed frequency ν and initial spread in Maxwellian β that the magnitude of the increase in normalized ion speed scales as

$$\Delta\rho_{eq} \propto \left(\frac{\varepsilon}{\delta}\right)^2. \quad (10)$$

And so, it can be seen that the magnitude of heating is a function of wave amplitude and frequency, which implies there is a way for manipulating the level of heating. It should be noted, however, that as these derivations were made for the small ε and β limits, non-linear effects may change this scaling relation for large wave amplitudes and low temperature plasmas

In sum, we have identified physical trends based on the analytical formulation presented in Eq. 8 that represent a promising possibility for controlling and ultimately optimizing the heating process. Before this optimization task can be undertaken, however, there is a need for experimental verification. And as the scope of the theory is limited by the assumptions outlined in the above section, the experiment must explore the predicted trends not just in the parameter space described above but outside it as well.

III. Testing Methodology, the Goal and the Approach

While experimental results have already confirmed that BEW can heat a plasma,⁴ we, with the goal in mind of optimizing the BEW heating process, seek to use the theoretical predictions from the previous section as a guideline for exploring the heating as a function of the defined non-dimensionalized parameters. Specifically, there are two experimental aims:

1. Investigate the parameter space outlined in the assumptions of Sec. II in an effort to verify the existence of an optimal frequency for heating and how the magnitude of heating scales with frequency and wave amplitude.
2. Characterize the physical process by which the beating wave heating mechanism produces plasma heating and use this understanding to optimize it.

For the first aim, in order to test the theoretical predictions outlined above, it is necessary to operate in the appropriate parameter space. Specifically, the experimental parameters must satisfy the assumptions about time scales and field amplitudes described in Sec. II. With respect to time scales, it has already been established that the plasma must be collisionless for the ions, $\omega_c \gg \nu_i$ where ν_i is the ion de-phasing collision frequency. Also, the heating time scale must be smaller than the collision time, $t_h \ll 1/\nu_i$. In order to simplify this expression, we see from Eq. 7 that the lowest frequency term is $\delta\omega_c$. Since δ is on the order of 0.1, we find that the equilibrated ion speed will only be achieved after averaging Eq. 7 over a sufficiently long time interval such that $t_h \gg 10/\omega_c$. By absorbing the factor of 10, we find that the heating restriction on time reduces approximately to the collisionless plasma condition. This condition can be achieved provided the correct ambient magnetic field and plasma density are selected.

The assumptions about electric and magnetic field magnitude correspond to the requirements that $\varepsilon < 1$ and $\beta < 0.1$. In order to find the appropriate parameter space for an experiment that will satisfy these criteria, we express them in terms of physical variables such that

$$E < \frac{\omega_c B}{k} \quad k_B T > \frac{5m\omega_c^2}{\pi k^2}. \quad (11)$$

We thus find restrictions on the initial temperature of the plasma, wave amplitude, and wavelength of exciting waves—all experimentally observed or controlled variables.

Once the criteria above are satisfied, the assertions from Sec. II are checked with three experiments:

1. Two beating electrostatic waves with frequencies differing by one cyclotron harmonic ($n = 1$) are introduced to the plasma, and a time-resolved measurement of the perpendicular ion velocity distribution function is performed with a Laser Induced Fluorescence (LIF) system. The evolution of the average perpendicular ion velocity is calculated from the changing distribution function and the increase over background inferred. The frequency of excitation is then increased by integer values so as to maintain the value of δ , and the procedure repeated. In this way, the frequency dependence of excitation is monitored.
2. The same procedure is followed as in 1; however, the value of δ is varied.
3. The frequency of excitation is fixed, and the wave amplitude is varied while the increase in average ion velocity is measured. The amplitude of waves in the plasma is remotely measured with the LIF system. This experiment establishes the dependence of the level of heating on wave amplitude.

The listed set of experiments from above will accomplish the primary goal of the investigation. For the second goal outlined above, a separate set of experiments seeks to fully characterize the heating mechanism. To this end, a spatially resolved LIF system is employed to measure the propagation of waves in the plasma and the average ion velocity throughout the test region. This provides insight into the wave-particle interaction intrinsic to BEW heating and provides a measure of the spatial extent of the effect.

IV. Experiment

A. Vacuum Chamber

A rendering of the experimental apparatus is shown in Figure 3. A single Pyrex cylinder 52" in length with a 6" inner diameter is placed concentrically in a 50" long, 10 ring solenoid. A small window placed at the end of the chamber provides longitudinal optical access while argon gas flows into the chamber through a feed in the cross at the opposite end of the chamber. A constant pressure of 0.1 to 30 mTorr is maintained by a 140 l/s turbo pump with a conductance controller as well as a roughing pump. Once the plasma is created, it propagates along the magnetic field lines into the experimental region. The solenoid is capable of producing a uniform 0.1 Tesla magnetic field along the central axis.

B. Plasma source

A Boswell type saddled antenna is placed around the vacuum chamber at one end of the solenoid. The antenna is actively water cooled and produces an inductive discharge by means of an ENI 13.56 MHz 1.28K power supply. The source is impedance matched to the plasma with an L network consisting of two Jennings 1000 pF 3kV variable vacuum capacitors. The antenna is positioned 18" away from the test region in order to minimize interference from the plasma source during ion heating measurements.

In order to minimize the ion-ion collisionality by reducing the plasma density, the antenna is operated at low power. While operating at low power also has the undesired effect of lowering the ionization fraction and therefore increasing the ion-neutral collision frequency, we have found that the ion-ion collision frequency is typically the larger contributor to the total de-phasing collision frequency. There is thus more gained from its reduction at low power. The dominance of ion-ion collisions can be seen in Figure 4 where we have shown the characteristic frequencies for a plasma in a 0.1 T magnetic field with the following parameters found in a typical 150-300 W inductive discharge: $n_e = 10^9 - 10^{10} \text{ cm}^{-3}$, $T_e = 3\text{eV}$, $T_i = 0.1\text{eV}$, and for low pressures, $< 3 \text{ mT}$, the ionization fraction α is $10^{-4} - 10^{-3}$.

C. Heating Antenna

A rendering of the the heating antenna is depicted in Figure 5. The antenna is a transverse Helmholtz configuration modeled after the geometry employed by Kline for single electrostatic wave heating.¹⁰ It is a 5" \times 9" rectangle and consists of 1/2" wide copper strip with 20 loops. The coils are operated in phase and powered with an ENI 100 W amplifier. The frequency of the amplifier is variable from 2 kHz to 2 MHz, which corresponds approximately to the range in normalized frequency of $\nu = 0.25 - 50$.

The antenna is mounted on an 18" carriage that can be rotated 180° and translated along the axis of the plasma. This is essential for spatially resolved ion temperature and wave measurements in the plasma.

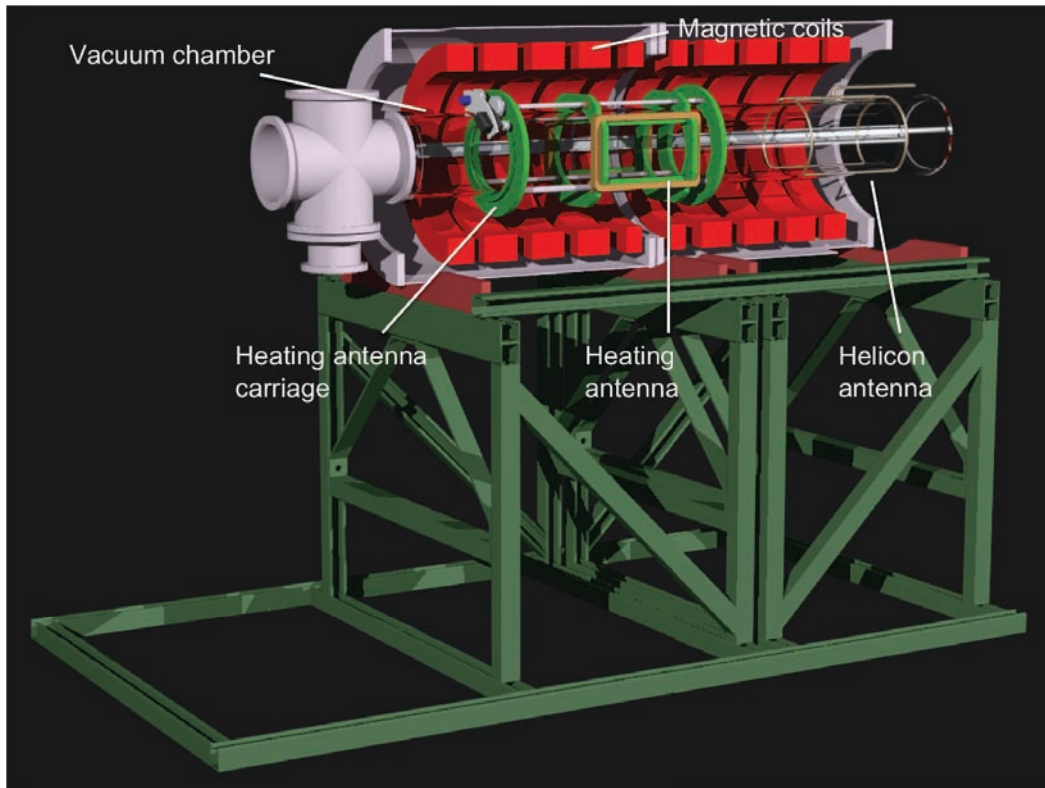


Figure 3. Rendering of the experimental apparatus.

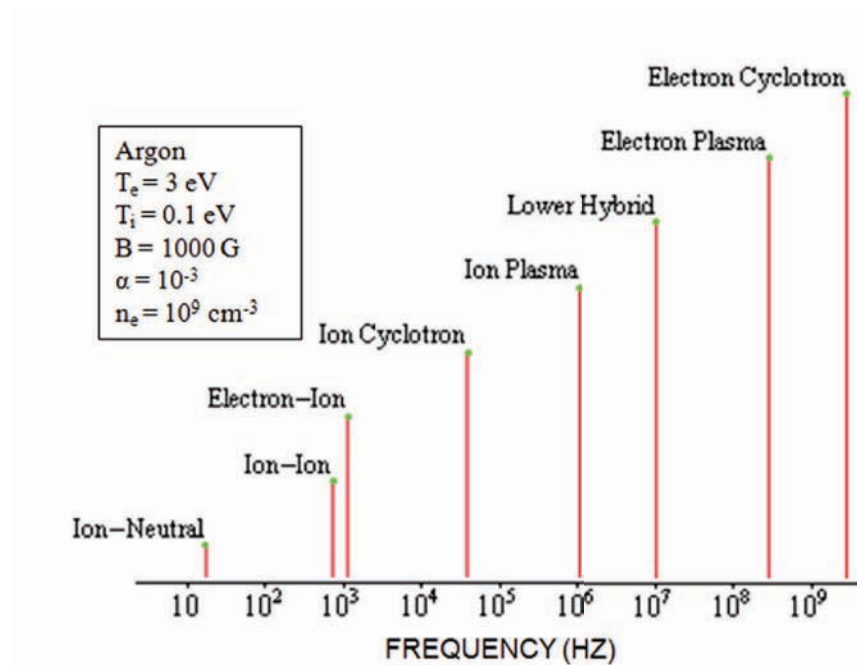


Figure 4. Typical frequencies for experimental plasma

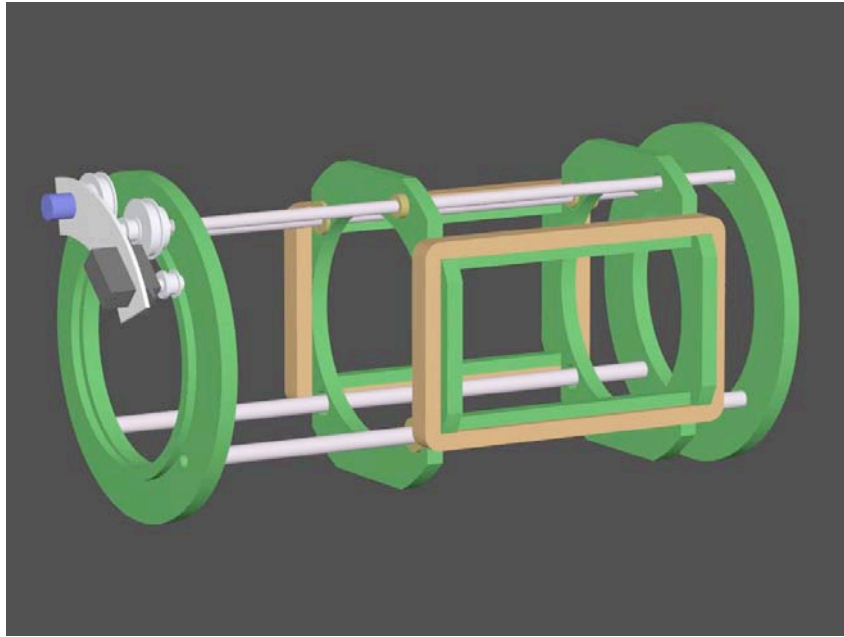


Figure 5. Computer rendering of the Helmholtz transverse antenna and its carriage.

Indeed, the LIF diagnostic necessary for these measurements is immobile and can only take perpendicular ion temperature and wave measurements at a fixed position in the axial direction and only along one diameter of the plasma (Figure 6). However, since the cylindrical plasma is assumed to be axisymmetric, by rotating the antenna around the center of the plasma, we effectively change the field of view of the LIF diagnostic. By that same token, the antenna can be translated so as to give a spatial measure of heating in the axial direction as well. The carriage is mounted flush to the vacuum chamber wall in order to provide optimal coupling between the antenna and plasma.

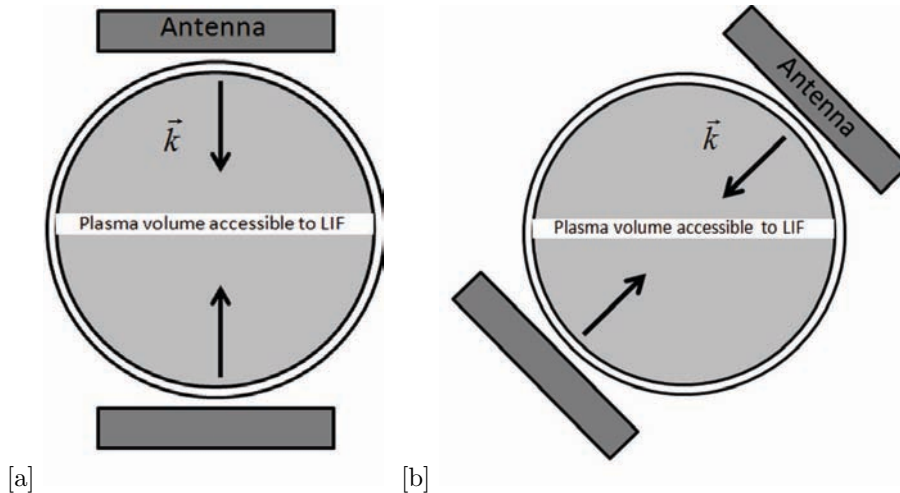


Figure 6. An axial cross-section of the experimental setup is shown. The light colored area represents the region optically accessible to the LIF system. The arrows indicate the direction of the launched waves. The rotation from the Figure a to Figure b, indicates how the mobile antenna can exploit the axisymmetry of the plasma to expand the optically accessible region of the LIF system.

The varying magnetic flux produced by the antenna generates electric fields in the plasma that in turn excite plasma waves. As has been observed in Ref. 10, the primary mode in the cyclotron frequency range excited by this technique is the forward branch of the Electrostatic Ion Cyclotron Wave (ESICW) with

dispersion relation given by¹¹

$$\omega^2 = \omega_c^2 + k^2 \frac{k_B T_e}{m} \quad (12)$$

where T_e is the electron temperature. It is noted that the ESICW is near-acoustic in nature and depends on the electron temperature. Also, there is minimal resonant damping that occurs. This feature makes the mode an ideal candidate for BEW heating investigations as it minimizes other heating effects that may be due to resonant absorption in the plasma.

D. Diagnostics

There are three major diagnostics in this experimental apparatus. First, an RF compensated Langmuir probe is employed for steady-state electron temperature and plasma potential measurements. Second, a low frequency directional coupler is placed in series with the ENI amplifier to give an indication of forward and reflected power from the heating antenna. Since the matching of the heating antenna to the plasma varies with frequency of excitation and plasma parameters, the directional coupler is a critical diagnostic for ensuring that comparable power levels are delivered to the plasma as the variables of the experiment are changed.

Finally, the primary diagnostic in this setup is the LIF system shown in Figure 7. The system has five components: a tunable diode laser centered at 668.6130 nm, a wavemeter, a signal chopper, a Stanford Lock-In Amplifier, and a collection optics lens stationed orthogonal to the incident laser beam. The central

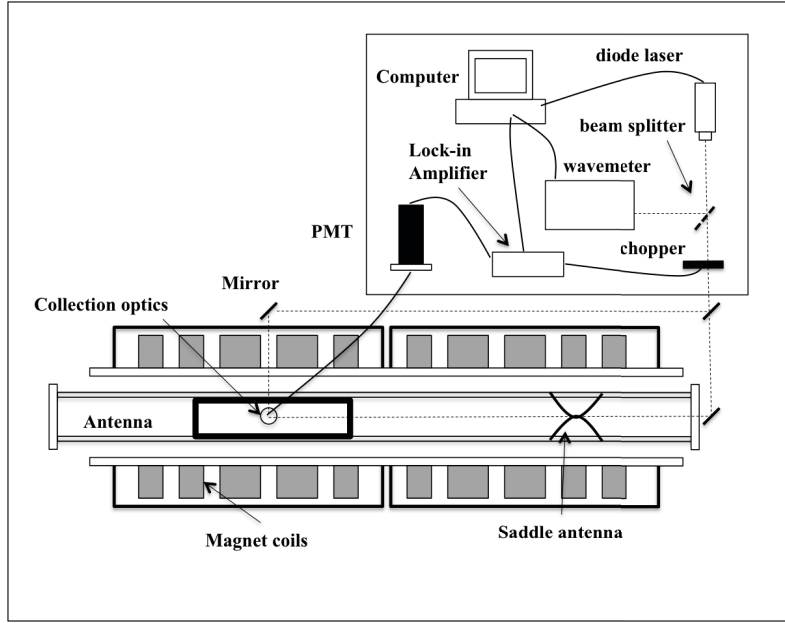


Figure 7. Schematic of the Laser Induced Fluorescence system. The dotted line indicates the laser beam path. There are two possible paths, parallel and perpendicular to the field lines for temperature and wave measurements in each direction. The system can only perform measurements in one direction at a time.

wavelength of the laser is tuned to the $3d^4F_{7/2} - 4p^4D_{5/2}^0$ transition of the metastable state of ArII which decays to the $4s^4P_{3/2}$ state producing 442.72 nm light. The wavelength of the beam is swept by 0.2 nm in the red and blue directions around the central wavelength, and the subsequent intensity profile of fluoresced light $I(\nu)$ is recorded. Due to the Doppler shift, the velocity distribution function $f(v)$ in the direction of the beam, denoted x' , can be shown to be proportional to the intensity of the fluoresced signal I such that

$$f(v_{x'}) \propto I\left(\nu_{l0} \left(1 - \frac{v_{x'}}{c}\right)\right), \quad (13)$$

where ν_{l0} is the central frequency of the laser and c is the speed of light. This normalized distribution of velocity $v_{x'}$ is for one direction in the plane of Larmor precession and can through the appropriate integration

of the intensity signal yield $v_{x'}^2$:

$$v_{x'}^2 = \frac{\int_{-\infty}^{\infty} v_{x'}^2 I(\nu_{l0}(1 - \frac{v_{x'}}{c})) dv_{x'}}{\int_{-\infty}^{\infty} I(\nu_{l0}(1 - \frac{v_{x'}}{c})) dv_{x'}}. \quad (14)$$

However, we need to know how the average perpendicular velocity varies $v_{\perp} = \sqrt{v_{x'}^2 + v_{y'}^2}$ where y' is the direction orthogonal to x' and in the plane normal to \hat{z} . If the assumption holds true that before the introduction of waves, the Larmor angles of precession are isotropic in phase, the LIF measurements in the x' direction can be assumed to be equivalent to measurements in the y' direction. Exploiting this fact, the perpendicular velocity evolution is found with the calculation

$$v_{\perp} = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{v_{x'}^2 + v_{y'}^2} I(\nu_{l0}(1 - \frac{v_{x'}}{c})) I(\nu_{l0}(1 - \frac{v_{y'}}{c})) dv_{x'} dv_{y'}}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(\nu_{l0}(1 - \frac{v_{x'}}{c})) I(\nu_{l0}(1 - \frac{v_{y'}}{c})) dv_{x'} dv_{y'}}. \quad (15)$$

In order to temporally resolve the evolution of the perpendicular ion speed distribution, the tunable laser is set at a fixed frequency while the heating antenna is triggered by a 100 ms pulse. The fluoresced signal after a fixed time delay is integrated over a 1 ms time interval. The process is repeated at a frequency of 1Hz and fed through a lock-in amplifier in order to eliminate ambient noise. Once this measurement is taken, the time delay between the start of the pulse and measurement is increased. After the duration of the pulse has been resolved for the given laser frequency, the frequency is incrementally changed and the process repeated. In this way, the intensity profile $I(\nu)$ can be constructed at each time step. Due to the limited temporal resolution of this method (1 ms), the de-phasing collision frequency must be sufficiently slow in order to yield the evolution of the velocity distribution in the absence collisions. As can be seen from Figure 4, such a collisionless environment ($\nu_{ii} < 10^3$, $\nu_{in} < 10^2$ where ν_{ii} and ν_{in} are the ion-ion and ion-neutral collision frequencies respectively) can be achieved at low densities, $n_e < 10^9$.

In addition to velocity distribution measurement, the wave properties including amplitude and dispersion relation are measured by triggering the lock-in amplifier to the frequency of the exciting waves. The methodology for this is discussed at greater length in Ref. 10. Furthermore, as described in Sec. C, we can find a spatial image of the propagation of waves in the plasma as well as a three dimensional image of ion temperature by rotating and translating the antenna.

E. Parameters of operation

The test of theoretical trends must, as discussed above, occur in the appropriate parameter space. The collision frequency for de-phasing collisions is given approximately by $\nu_i = \nu_{ii} + \nu_{in}$ where we have neglected electron-ion collision frequencies since they produce minimal momentum change for the ions. By comparing ν_i to the cyclotron frequency ω_c , we can see from Figure 4 that the collisionless criteria for our plasma is satisfied.

As for appropriate electric field, we can substitute in the dispersion relation from Eq. 12 into the criteria outlined in Eq. 11 to find

$$E < \sqrt{\frac{B^2 k_B T_e}{mq^2(\nu^2 - 1)}} \quad \nu^2 - 1 > \frac{5T_e}{\pi T_i}. \quad (16)$$

Using the representative plasma parameters depicted in Figure 4, we find the following physical constraints

$$E < \frac{25000}{\nu^2 - 1} \frac{V}{m} \quad \nu > 7. \quad (17)$$

For a 0.1 Tesla magnetic field, our ENI amplifier is capable of producing $0.25 < \nu < 50$, so the second condition is easily satisfied. However, the amplitude of the waves must be carefully controlled in order to operate in the correct parameter space.

V. Conclusions

We have presented the design and implementation of an experiment dedicated to investigating the optimal frequency and scaling relations predicted for BEW plasma heating. This experiment employs an RF plasma source that generates a low density plasma in which beating electrostatic waves are launched by means of a transverse Helmholtz antenna. We described the theoretical basis for the predicted optimal frequency and scaling relations for heating as well as the methodology for testing these trends. We also provided a description of the diagnostics and analysis techniques and estimated the appropriate frequency and electric field ranges to test the parameter space described in the theoretical derivations. We finally discussed a secondary set of experiments for measuring the spatial extend of the heating and wave-particle interaction.

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